# Application of the Adjoint Method for Vehicle Aerodynamic Optimization

Dr. Thomas Blacha, Audi AG

GoFun, Braunschweig 22.3.2017

# Outline

**Adjoint method in theory**

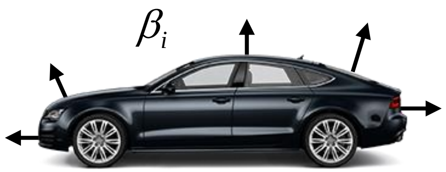
**Adjoint method in vehicle aerodynamics**

- Necessary simplifications
  - Transient vs. steady state
  - RANS => (D)DES
- Test case: Audi Q5 (built in 2012)
  - Required accuracy for primal solution
  - Convergence of the adjoint method
  - Consistency => accuracy of adjoint predictions
  - Results on the Audi Q5 built in 2012

**Conclusions**

# Adjoint method in theory

## Minimization problem:




minimize $F_d = F_d(p, \vec{v}, \beta)$ with $\vec{\Re}(p, \vec{v}, \beta) = 0$

$$\boxed{\begin{aligned}(\vec{v}\cdot\nabla)\vec{v} + \nabla p - \nabla\cdot(2\nu D(\vec{v})) = 0 \\ \nabla\cdot\vec{v} = 0\end{aligned}}$$

Lagrange multipliers

$\Rightarrow$ Lagrange problem: $L = F_d + \int_\Omega (\vec{u}, q)\vec{\Re}\, d\Omega$

$\Rightarrow$ Total variation: $\delta L = \delta_\beta L + \cancel{\delta_v L + \delta_p L}$ $\Rightarrow$ $\delta_v L + \delta_p L \overset{!}{=} 0$

$$\boxed{\text{Adjoint equations for } \vec{u}\ \&\ q}$$

$\Rightarrow$ Sensitivity: $G_i = \dfrac{\partial L}{\partial \beta_i} = -\nu \dfrac{\partial u_t}{\partial n_t} \cdot \dfrac{\partial v_t}{\partial n_t}\Big|_i$

---

primal transport equations

$$\underbrace{(\vec{v}\cdot\nabla)\vec{v}}_{\text{convection}} = \underbrace{-\nabla p}_{\text{pressure}} + \underbrace{\nabla\cdot(2\nu D(\vec{v}))}_{\text{diffusion}}$$

$$\nabla\cdot\vec{v} = 0$$

adjoint transport equations

$$\underbrace{-(\vec{v}\cdot\nabla)\vec{u}}_{\text{backward convection}} - \underbrace{\nabla\vec{u}\cdot\vec{v}}_{\text{transpose convection}} = \underbrace{-\nabla q}_{\text{adjoint pressure}} + \underbrace{\nabla\cdot(2\nu D(\vec{u}))}_{\text{adjoint diffusion}}$$

$$\nabla\cdot\vec{u} = 0$$

Influences significantly
- Convergence behavior
- Applicable schemes
- Accuracy

# Adjoint method in vehicle aerodynamics

## Simplifications

transient Spalart-Allmaras DES
$\Rightarrow$ time averaging of p(t) and v(t)

$\bar{p}$ und $\bar{v}$

RANS* Spalart-Allmaras transport equation
$\Rightarrow \nu_t$

*transient adjoint method is much too expensive!

$\bar{p}$ & $\bar{v}$

$\nu_t$

**„primal solution“**

$\bar{p}$, $\bar{v}$ und $\nu_t$

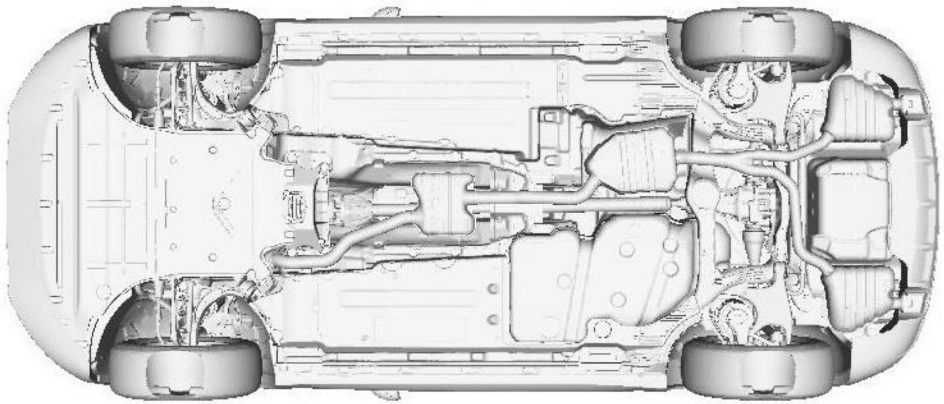
**steady state adjoint**
(frozen turbulence)

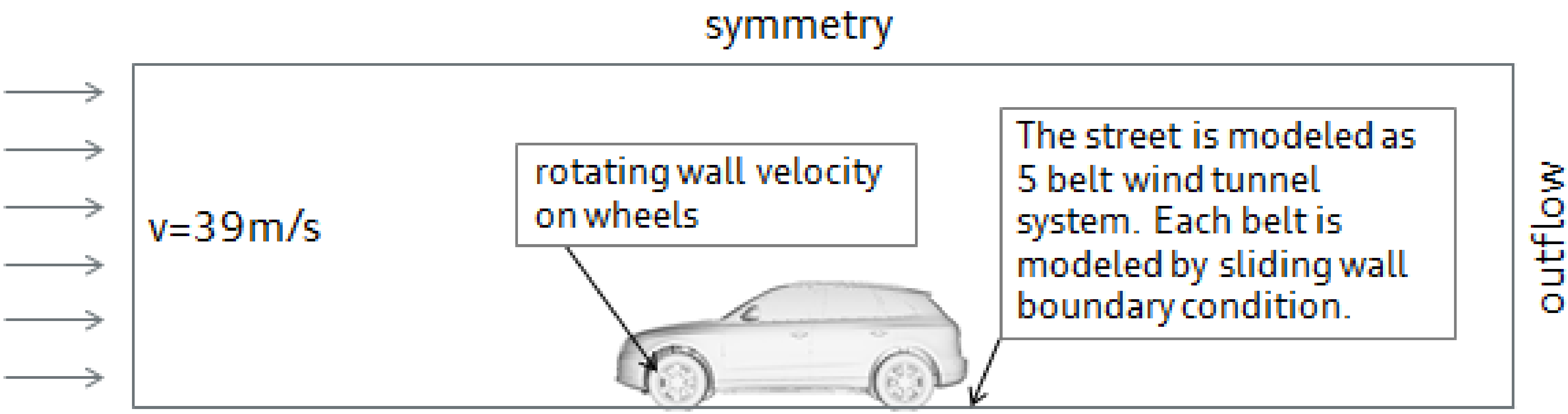
interpretation of sensitivities during early optimization at 1:4 model scale

$\Rightarrow$ FKFS
(no automated morphing)

# Adjoint method in vehicle aerodynamics

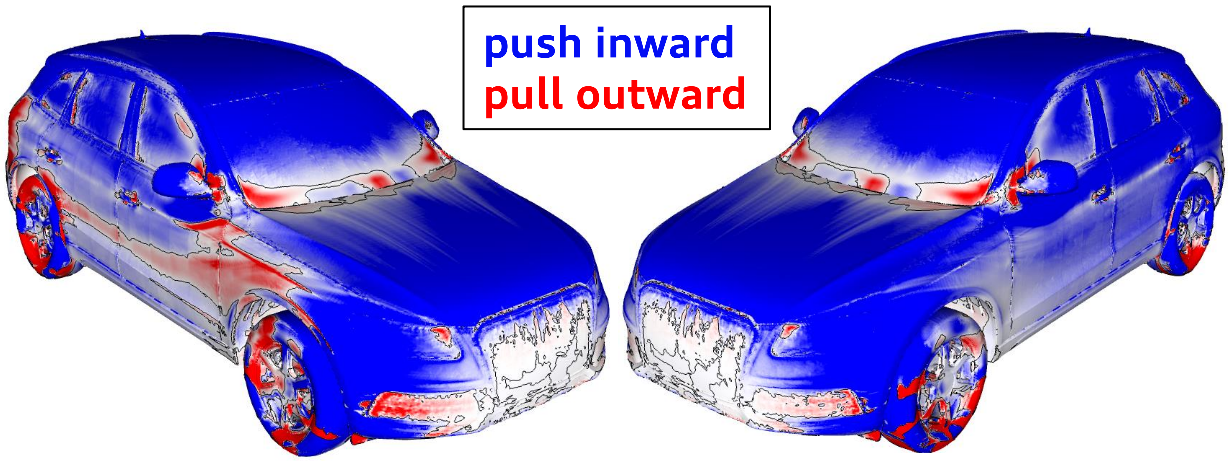
## Test case: Audi Q5 (built in 2012)





# Adjoint method in vehicle aerodynamics

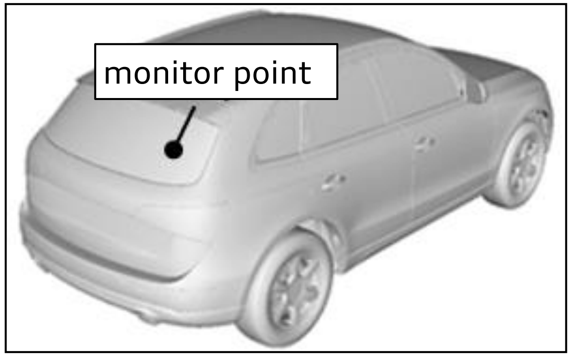
## Observations

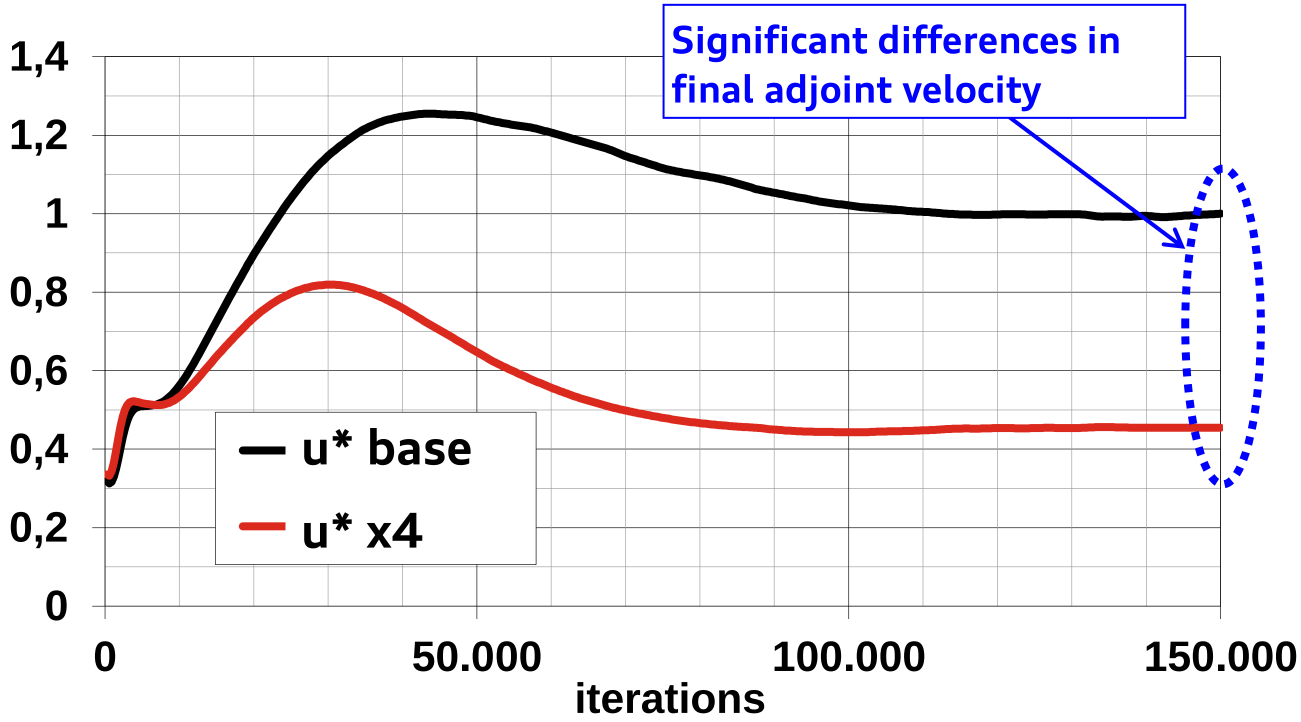



unphysical asymmetry

Asymmetry is very sensitive with respect to convergence level of $\nu_t$ !

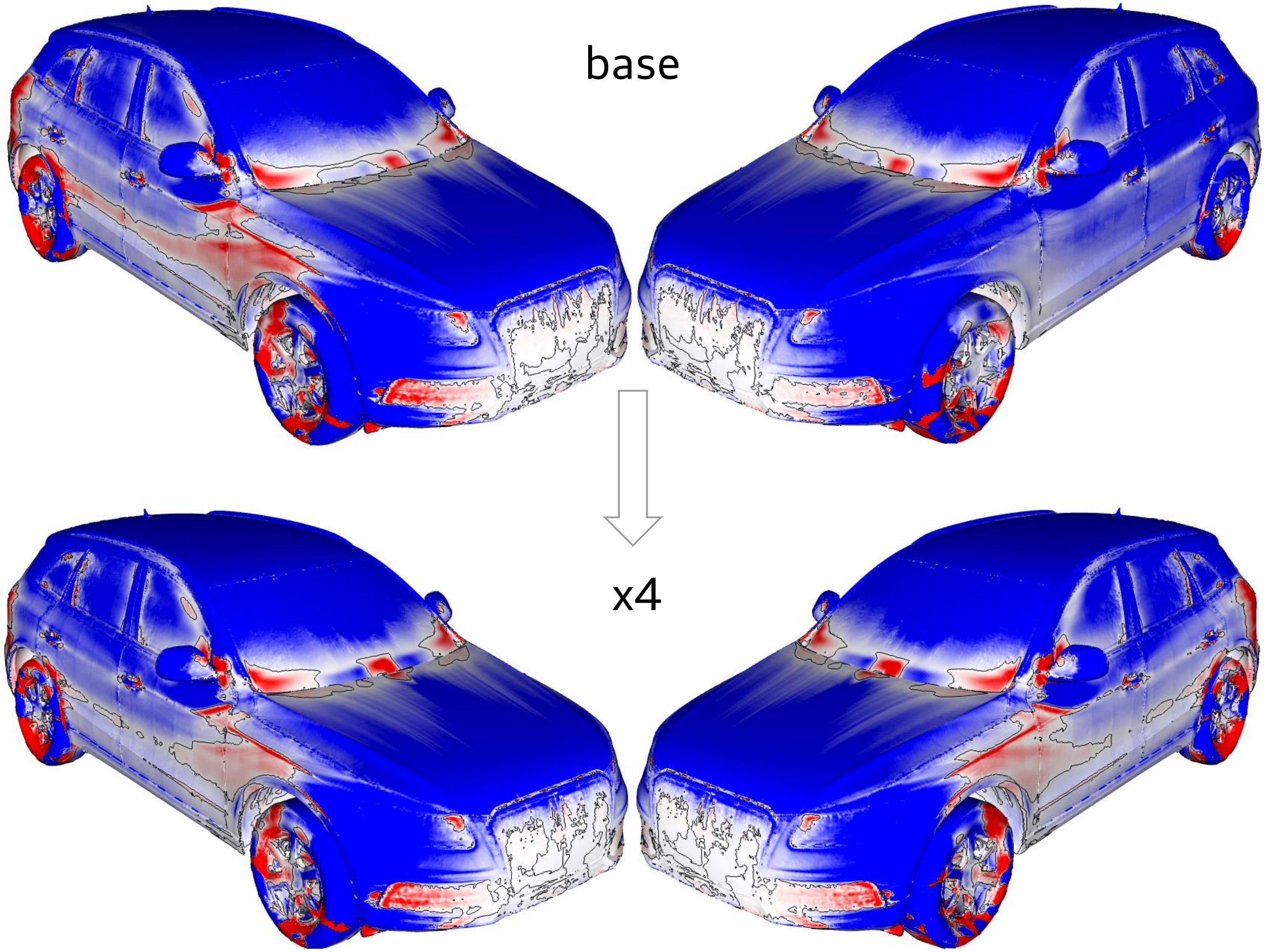
Better convergence quality of primal fields v & p necessary?!
⇒ Longer time averaging
⇒„x4“



# Adjoint method in vehicle aerodynamics

## Required accuracy for primal solution

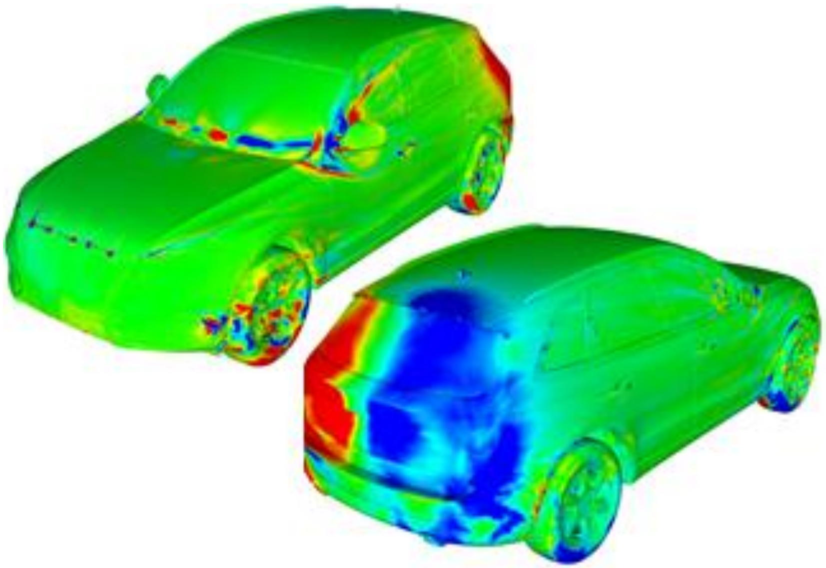



⇒ Symmetry significantly improved

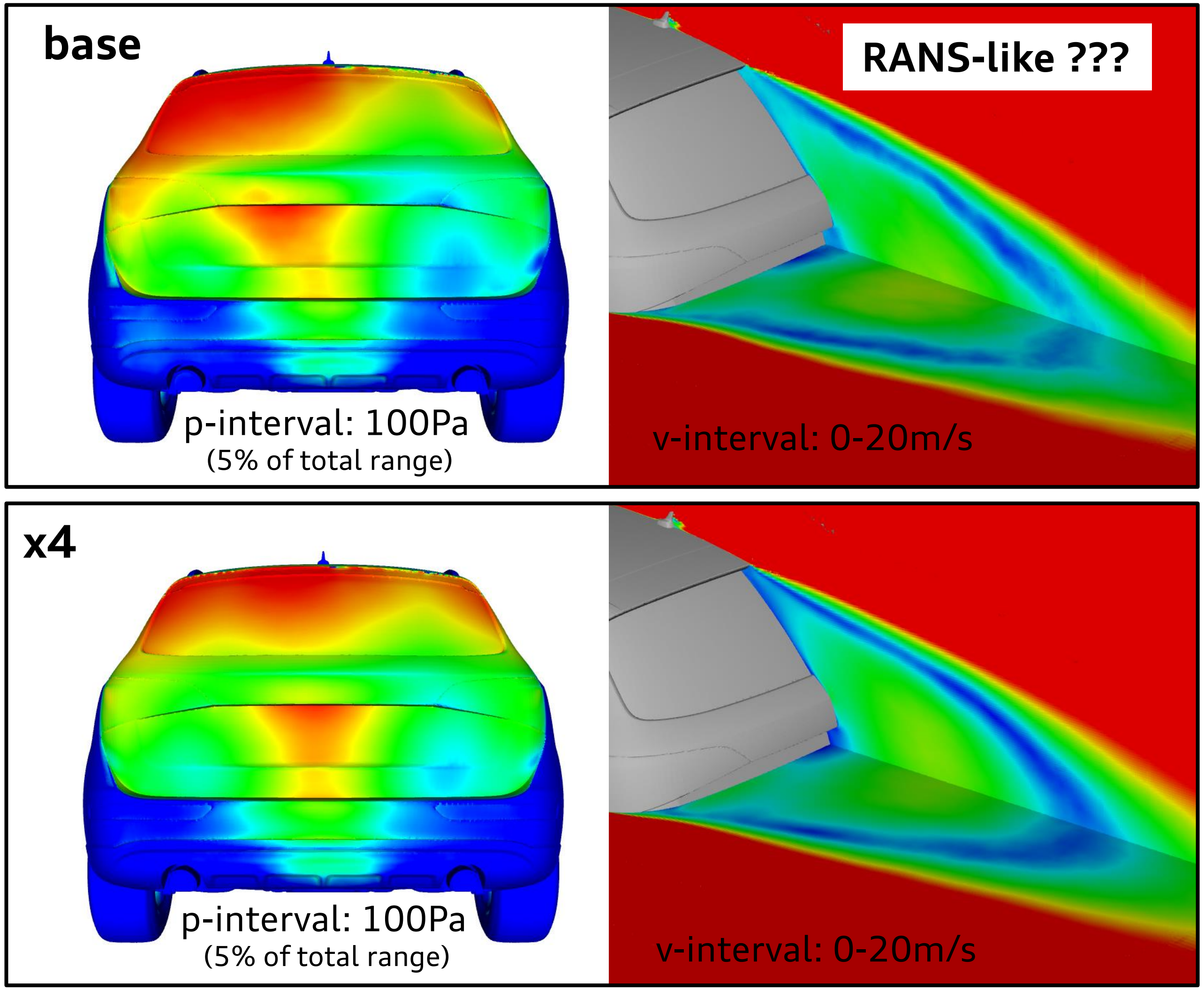
# Adjoint method in vehicle aerodynamics

## Required accuracy for primal solution

$p_{base} - p_{x4}$
p-interval: 20Pa

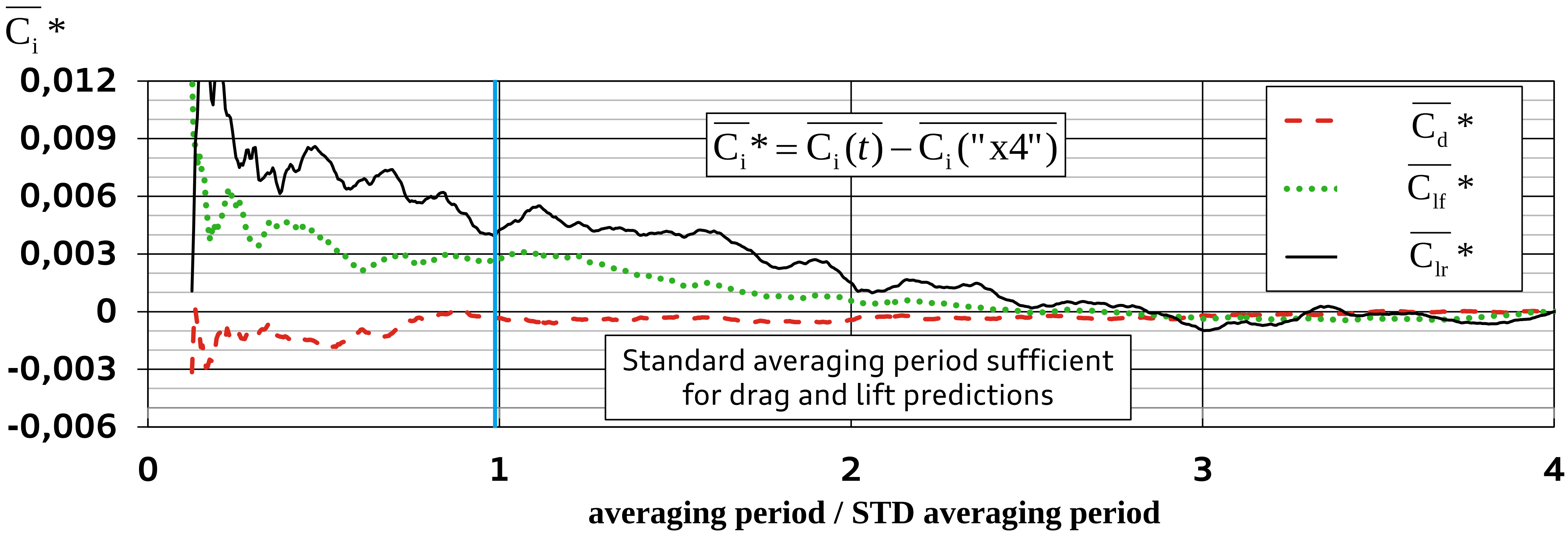


⇒ **Main differences in the rear of the car (wake region)**



# Adjoint method in vehicle aerodynamics

## Influence of time averaging period




⇒ Different treatment of primal solution necessary if adjoint calculation is planed!

| **Non-adjoint** | ⇔ | **Adjoint** |
|---|---|---|
| Accurate scale resolving methods (small time period) | | Methods, which resolve only the most important turbulent structures |

# Adjoint method in vehicle aerodynamics

## Consistency

Time averaged DES values and RANS turbulent viscosity

$\overline{v}$ , $\overline{p}$ & $\nu_t$

⇩

Input values for RANS adjoint solver

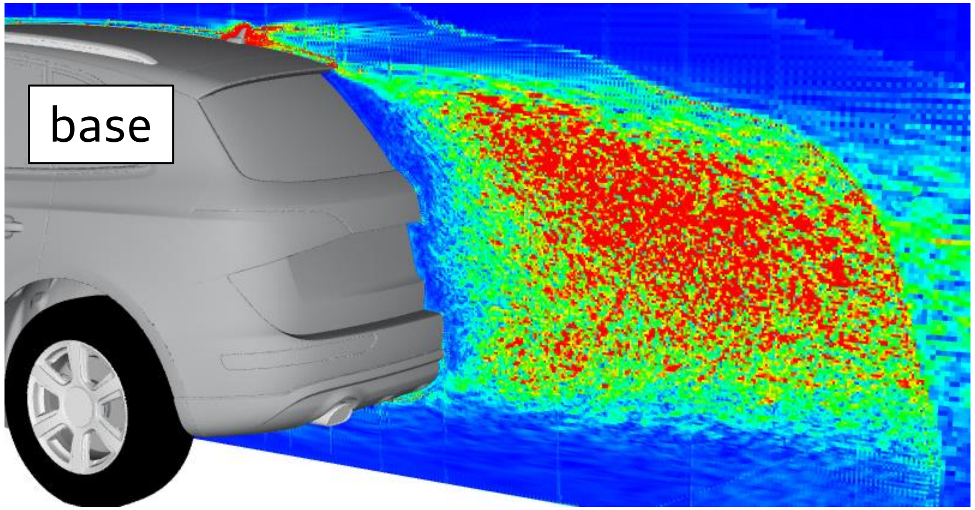
⇩

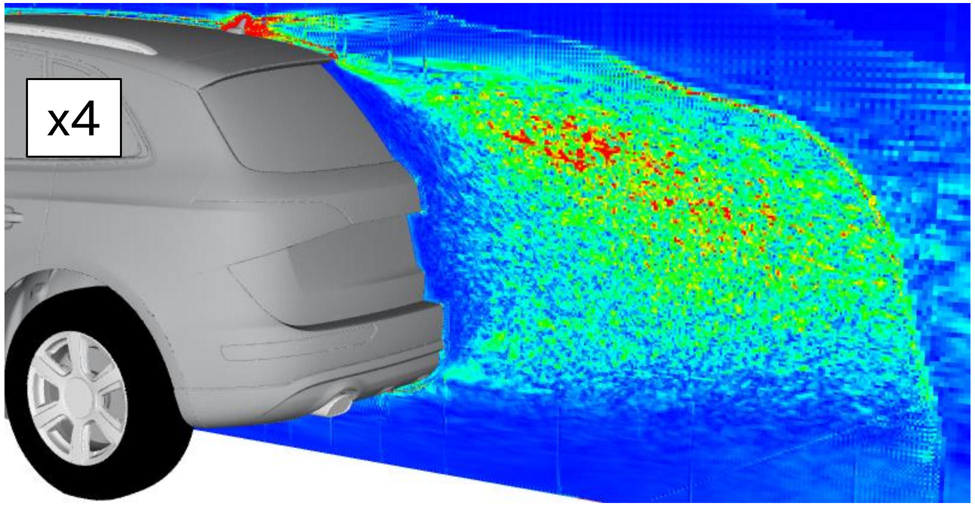
**continuity**
(mass conservation is linear) √

**momentum** X

(highly non-linear)
RANS turbulent viscosity can only represent isotropic turbulence.

Residual of RANS momentum equation




RANS momentum residual falls

# Adjoint method in vehicle aerodynamics

## Consistency

**Predictions by means of time averaged transient primal (e.g. PANS, DES, etc.) and steady state adjoint.**

Primal input values: $\overline{v}$, $\overline{p}$ und $\nu_t$

**How to obtain $\nu_t$ ?**

$$\vec{\Re}_{\vec{v}} = (\vec{v}\cdot\nabla)\vec{v} + \nabla p - \nabla\cdot(2\nu D(\vec{v})) \overset{!}{=} 0$$

influence on adjoint

$$-(\vec{v}\cdot\nabla)\vec{u} - \nabla\vec{u}\cdot\vec{v} = -\nabla q + \nabla\cdot(2\nu D(\vec{u}))$$

**Definition/Meaning of $\nu=\nu_m+\nu_t$ ?!**

$\nu_t$ is pure turbulent RANS viscosity => calculated using RANS turbulence model

$\Longrightarrow$ Velocity field of primal has to fulfill eddy viscosity law

$$\overline{v_i' v_j'} = -\nu_t\left(\frac{\partial\overline{v_i}}{\partial x_j} + \frac{\partial\overline{v_j}}{\partial x_i}\right) + \frac{2}{3}k\delta_{ij}$$

$\Longrightarrow$ Only isotropic turbulence can be represented

$\nu_t$ is closure value of any meaning

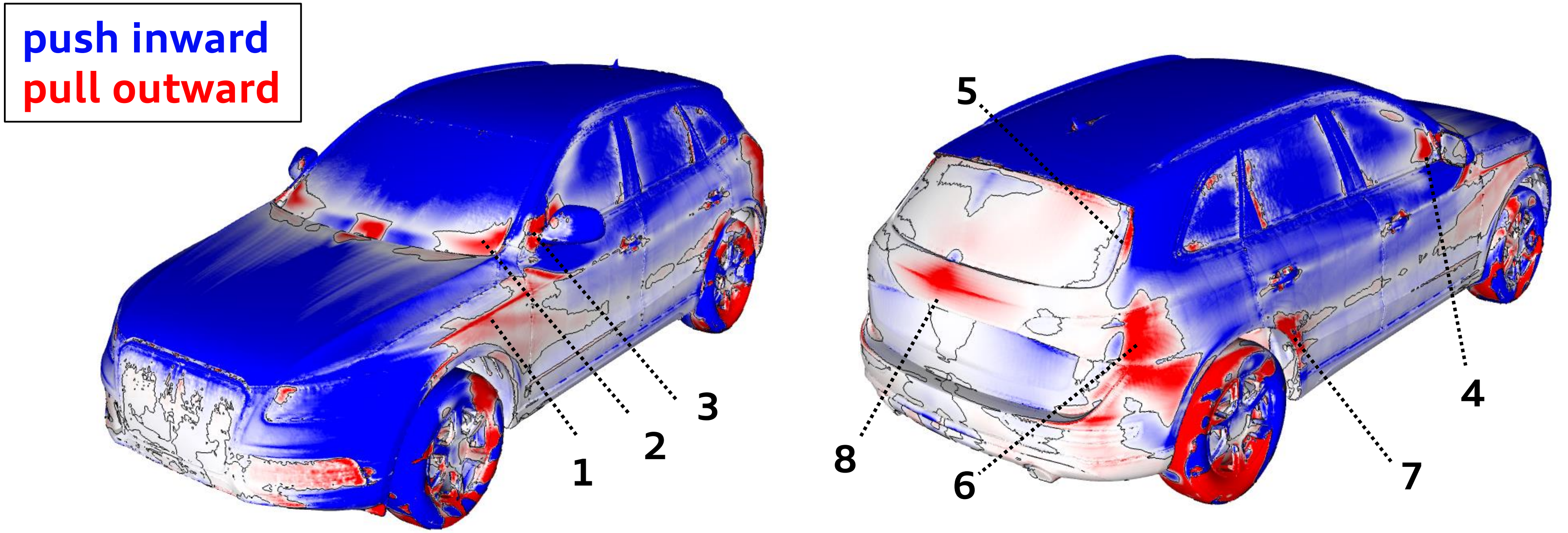
$\Longrightarrow$ Can be chosen in order to minimize the residual.

(Does not necessarily require to solve a $\nu_t$ transport equation)

$\Longrightarrow$ Not necessarily of scalar type => tensorial viscosity also possible

# Adjoint method in vehicle aerodynamics

## Results on Audi Q5 (built in 2012)




| | ΔCd | |
|---|---|---|
| **Measure** | **CFD** | **1:4 Exp. FKFS** |
| 1: Lateral kink with bigger radius | – | -0.0005 |
| 2: More material at bottom of front window near A-pillar | – | -0.001 |
| 3: More material in front of side view mirror | – | -0.001 |
| 4: Extension of mirror base by 110mm | -0.002 | -0.002 |
| 5: Sharper trailing edge on D-pillar | -0.002 | -0.004 (also 1:1) |
| 6: Outward pulling with sharp trailing edge on rear shoulder | -0.003 | -0.001 |
| 7: Outward pulling before rear wheel | - | 0 |
| 8: Mounting of a small horizontal plane below rear window | -0.004 | +0.004 |

# Conclusions

- Greatest challenges for the application of the adjoint method in vehicle aerodynamics are
  - Convergence (final convergence, numerical noise)
  - Accuracy (scheme order, influence from primal solution, discretization of ATC, limiting)
  - Computational cost for transient adjoint method still too high

  $\Longrightarrow$ Strategies necessary which rely on steady state adjoint
- If time averaged flow fields are used as input values for a steady state adjoint solver, consistency cannot be guaranteed. In particular, the reliability of predicted sensitivity maps significantly depends on the choice of time averaging window. The required minimum time averaging period is in general significantly larger than for regular drag and lift predictions.
  - Adjoint and Non-Adjoint setup necessary for primal solution!
  - Alternative calculation of $\nu_t$ in order to improve consistency (There are limits!) ?
- Nevertheless stable results were obtained even on an automatically generated unstructured grid. The predicted influence of different measures on drag is in good agreement with measurements at 1:4 model scale and with time averaged DES calculations.

## References

Blacha, T., Gregersen, M.M., Islam, M. and Bensler, H., “Aerodynamic Vehicle Optimizations Using the Continuous Adjoint Method”, *Proceedings of the 10th FKFS-Conference*, 2015

Blacha, T., Gregersen, M., Islam, M., and Bensler, H., "Application of the Adjoint Method for Vehicle Aerodynamic Optimization," SAE Technical Paper 2016-01-1615, 2016, doi:10.4271/2016-01-1615.

Othmer, C. A continuous adjoint formulation for the computation of topological and surface sensitivities of ducted flows. *Int. J. Num. Meth. Fluids* 58, 861 (2008).

# Thank you!

# Questions?