Universität Rostock Traditio et Innovatio

Methods for Generating Turbulent Inflow Boundary Conditions for LES and DES

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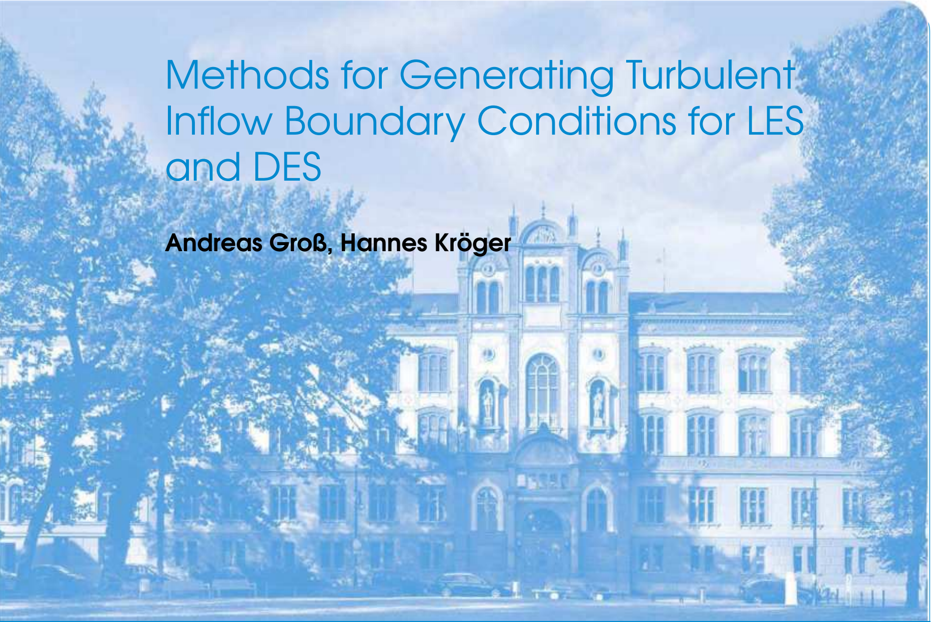
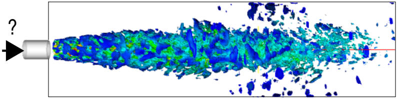


Contents

Motivation

- For a detailed analysis of unsteady flows, a partial ($\Rightarrow$DES[1], LES[2]) or full ($\Rightarrow$DNS[3]) resolution of the turbulent motions is necessary
- DES and LES find already widespread application in industrial practice
- There is often already a turbulent flow at the inflow boundaries. A spatially and temporally resolved turbulent velocity field has to be prescribed there: [4]

$$\vec{U}(\vec{x}, t) = \langle \vec{U} \rangle (\vec{x}) + \vec{u}(\vec{x}, t)$$

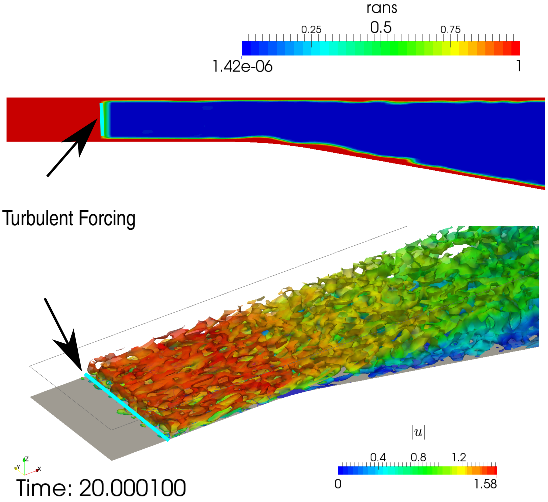


$\Rightarrow$The velocity fluctuations have to be generated somehow

[1] Detached-Eddy Simulation
[2] Large-Eddy Simulation
[3] Direct Numerical Simulation
[4] temporal average $\langle \phi \rangle = \int_{-\infty}^{\infty} \phi dt$

Motivation II

- DES: Turbulence insertion triggers RANS-to-LES transition
- Synthesis of realistic turbulent structures is required



Requirements

$$\vec{U}(\vec{x},t) = \langle\vec{U}\rangle(\vec{x}) + \vec{u}(\vec{x},t)$$

In order to act as turbulence, the generated velocity fluctuations need to fulfill a number of properties:

1. Reynolds Stresses, Amplitude: $\langle u_i u_j \rangle(\vec{x})$
2. Spatial Correlation, i.e. spectrum: $R_{ij}(\vec{x},\vec{\eta}) = \frac{\langle u_i(\vec{x},t) u_j(\vec{x}+\vec{\eta},t)\rangle}{\langle u_i(\vec{x},t) u_j(\vec{x},t)\rangle}$
3. out of it, the length scales follow: $L_{ij}(\vec{x},\vec{e}_\eta) = \int\limits_0^\infty R_{ij}(\vec{x},\eta\vec{e}_\eta)d\eta$
4. Continuity constraint $\nabla \cdot \vec{u} = 0$

   $\Rightarrow$violation may lead to numerical problems and/or parasitic pressure fluctuations

Methods

Possible approaches for artificial generation of turbulent fluctuations:

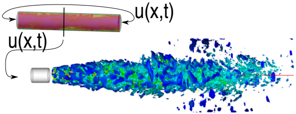
1. Random Velocity Fluctuations

in every discrete point with specified amplitude.

- Fluctuations are spatially and temporally uncorrelated
  $\Rightarrow L_{ij} = 0$
- Divergence-free constraint is violated, strong damping of fluctuations downstream
  $\Rightarrow$Unfeasible

2. Precursor-Simulation

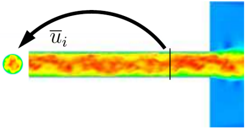
Generation of turbulent fluctuations by auxiliary simulation with periodic boundary conditions.



- Computationally expensive
- Restricted to simple, generic flows

3. Recycling-Method

Extension of domain upstream and extraction of turbulent velocities from the interior domain.



- Increase of computational expense
- Only applicable to fully developed, stationary flows (tube or channel flow)

4. Synthetic Turbulence Generation

Multiple methods have been proposed:

- Sinusoidal Modes by Kraichnan
  - Superposition of global basis functions
  - Disadvantage: inhomogeneous statistics difficult
- Digital filtering method by Klein
  - Disadvantage: constant time step and grid required
  - reproduction of autocorrelation function difficult
- Superposition of “turbulent“ structures $\Rightarrow$structure based methods

The Turbulent Spot Method

- a.k.a Synthetic-Eddy-Method (SEM)
- superposition of a number of local, compact velocity fields (the ”Turbulent Spots“) on a mean velocity field.

Thereby:

- The structures are randomly distributed
- They are convected by the mean velocity through the inflow boundary
- The inner velocity distribution is scaled by a random parameter
- The size of the structures controls the length scale

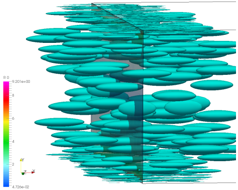


Figure: Snapshot of a population of turbulent structures around the inlet

Development Status

- The Structure Based Method is developed at LEMOS since 2003
- parallel developments at Manchester University (SEM)
- Realized features so far:
  - Prescribed inhomogeneous and anisotropic Reynolds Stresses
  - Prescribed energy spectrum and length scale (inhomogenenous, but isotropic $L_x = L_y = L_z$)
  - Not strictly divergence-free [5]
- Developments in the current project:
  - Prescribed length scale (anisotropic)
  - Strictly divergence-free
  - Extensive validation

[5] because of Cholesky transformation

Inner Velocity Distribution

- The inner velocity distribution $\vec{u}$ determines the autocorrelation function (= energy spectrum):

$$R_{ij}(\vec{x},\vec{\eta}) = \frac{\iiint\limits_{(V)} u_i(\vec{x})u_j(\vec{x}+\vec{\eta})dxdydz}{\iiint\limits_{(V)} u_i(\vec{x},t)u_j(\vec{x},t)dxdydz}$$

- The decay distance of the autocorrelation function determines the length scales

$$L_i(\vec{x}) = \int\limits_0^{\infty} R_{ij}(\vec{x},\eta\vec{e}_i)d\eta$$

$\Rightarrow$Velocity distribution inside the spots determines energy spectrum and thus length scales

Scaling of the Fluctuations

If the algorithm produces fluctuations with unit amplitude:

1. $\langle u_{ij}^2 \rangle = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$
2. $\langle u_i \rangle = 0$

the Reynolds Stresses can be conditioned by a Cholesky transform::

$$\vec{U} = \langle \vec{U} \rangle + \mathbf{A}\vec{u}$$

with the matrix

$$\mathbf{A} = \begin{pmatrix} \sqrt{R_{11}} & 0 & 0 \\ R_{21}/A_{11} & \sqrt{R_{22} - A_{21}^2} & 0 \\ R_{31}/A_{11} & (R_{32} - A_{21}A_{31})/A_{22} & \sqrt{R_{33} - A_{31}^2 - A_{32}^2} \end{pmatrix}$$

$\Rightarrow$But: Cholesky transform deteriorates second-order statistics and continuity properties of generated turbulence

Implementation

- The Structure Based Method was implemented in OpenFOAM ("InflowGenerator BC")
- Tasks in this context:
  - scaling the fluctuation amplitudes to fulfill Reynolds Stresses
  - placement of the spots

Thereby:

- Produce structures online, no preprocessing steps
- Avoid numerical calibration of statistics
- Prescribe as a spatial field (inhomogeneous!):
  - length scale
  - the Reynolds Stresses

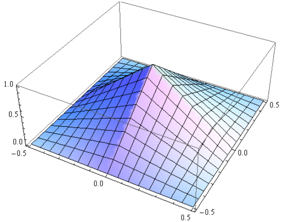
Types of Turbulent Structures

1.Spots

Direct ansatz functions for velocity

Hat Spot

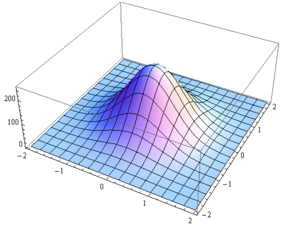
- velocity distribution: tent function



- Simple
- RS scaling by Cholesky transform
- Violates continuity constraint
- Unphysical energy spectrum

Gaussian Spot

- velocity distribution: gaussian function



- RS scaling by Cholesky transform
- Violates continuity constraint
- Energy spectrum of decaying turbulence

2. Vortons

Velocity distribution derived from vector potential $\vec{A}$:

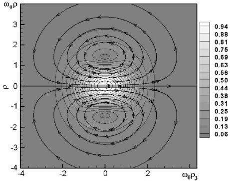
$$\nabla \cdot \vec{u} = \nabla \cdot (\nabla \times \vec{A}) = 0$$

Isotropic Vorton

- symmetric velocity distribution

$$u_z = u_r = 0$$

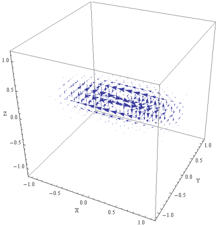
$$u_\theta = \pi \sqrt{\frac{2C_E}{C_1}} k_0^5 \exp(-k_0^2 r^2/2) r$$



- RS scaling by Cholesky transform
- Fulfills continuity constraint only if RS are isotropic
- Energy spectrum of decaying turbulence

New: Anisotropic Vorton

- Most recent development
- unsymmetric velocity distribution from transformed vector potential



- No Cholesky transform required
- Fulfills continuity constraint always
- Unphysical energy spectrum

Anisotropic Vorton

- The internal velocity field follows from the vector potential $\vec{A}$:

$$\vec{u} = \nabla \times \vec{A}$$

For isotropic turbulence, a symmetric vector potential $\vec{A} = A(r)\vec{e}_z$ was derived earlier (Kornev 2007[6]).

- To account for anisotropy, scaling parameters are introduced into the vector potential:

$$\vec{A} = \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)\right] \begin{pmatrix} x\gamma_x \\ y\gamma_y \\ z\gamma_z \end{pmatrix}$$

- with the free parameters $\gamma_x, \gamma_y, \gamma_z, \sigma_x, \sigma_y$ and σ_z.

[6] **kornev_divergence_free_vortons**.

Anisotropic Vorton

- Closed expressions for the Reynolds stresses can be derived:

$$\mathbf{R} = \frac{\pi^{3/2}}{4} \begin{pmatrix} \frac{\sigma_x(\gamma_y\sigma_y^2 - \gamma_z\sigma_z^2)^2}{\sigma_y\sigma_z} & 0 & 0 \\ 0 & \frac{\sigma_y(\gamma_x\sigma_x^2 - \gamma_z\sigma_z^2)^2}{\sigma_x\sigma_z} & 0 \\ 0 & 0 & \frac{\sigma_z(\gamma_x\sigma_x^2 - \gamma_y\sigma_y^2)^2}{\sigma_x\sigma_y} \end{pmatrix}$$

- also for the length scales

$$L_i = \int_0^\infty r_{ii}(\eta\vec{e}_i)\mathrm{d}\eta = \sqrt{\pi}\sigma_i$$

for $i = x, y, z$.

$\Rightarrow$ From prescribed Reynolds stresses and length scales, the free parameters can be deduced.

Anisotropic Vorton

- But: not all combinations of length scales can be reproduced. A solution requires:

$$\pm\sqrt{R_2\frac{L_1L_3}{L_2}} \pm \sqrt{R_3\frac{L_1L_2}{L_3}} = \pm\sqrt{R_1\frac{L_2L_3}{L_1}}$$

or

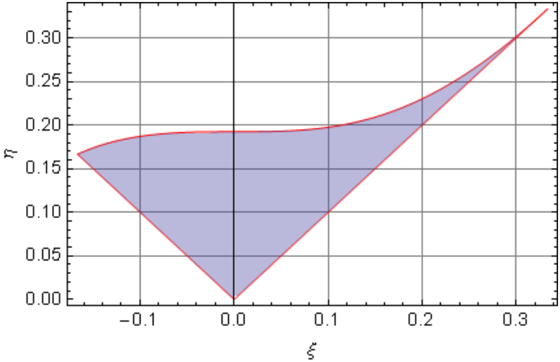
$$L_3 = \frac{\pm L_1L_2\sqrt{R_3}}{\pm L_2\sqrt{R_1} \pm L_1\sqrt{R_2}}$$

$\Rightarrow$Prescription of two length scales determines the third.

- Especially in the case of isotropic turbulence $R_1 = R_2 = R_3 = R$ with $L_1 = L_2 = L$ the condition $L_3 = L/2$ needs to be fulfilled.

Realizable Anisotropy States

Lumley-Triangle

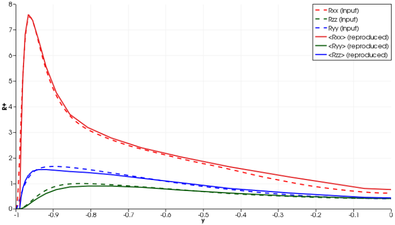


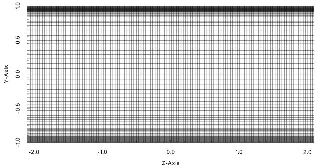
- shaded area is reproducable by ansiotropic vortons

  ⇒entire area

Reproduction of Reynolds stresses

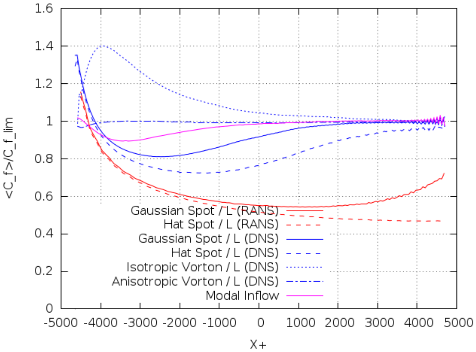
Reproduction of Reynolds stresses in LES of channel flow at $Re_\tau = 395$.

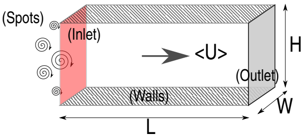




- Dashed: prescribed input profiles
- Solid: reproduced profiles

Channel Flow - Adaption Length





Quantity	Value
H	2
W	4.19
L	24
Re_τ	395

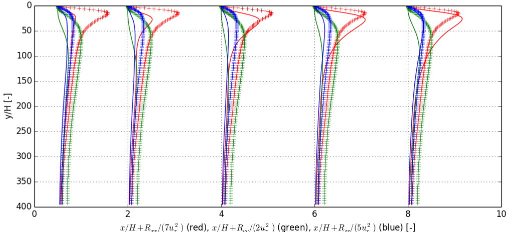
Red: isotropic length scales Blue: anisotropic length scales

$\Rightarrow$

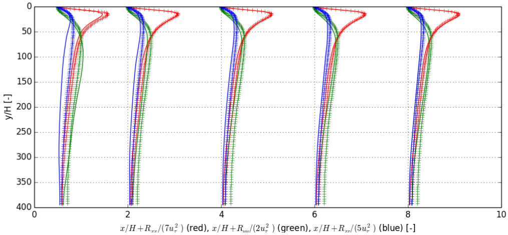
Velocity distribution shape has big influence on adaption length, influence of of length scales anisotropy is remarkable as well

Reynolds Stresses

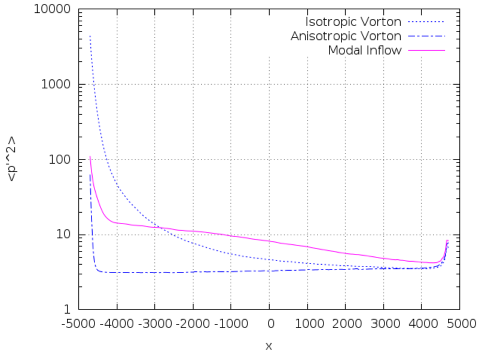
hatSpot / L(DNS)

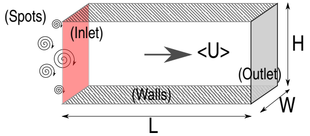


anisotropic Vorton / L(DNS)



Parasitic Pressure Fluctuations

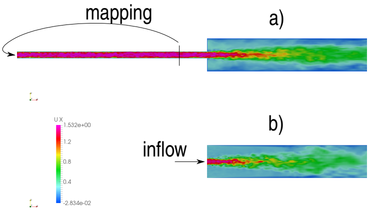


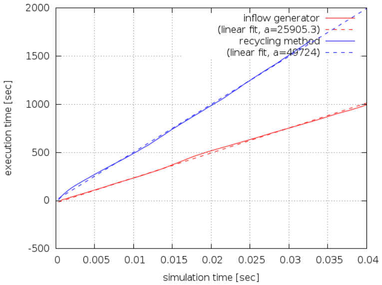


Quantity	Value
H	2
W	4.19
L	24
Re_τ	395

⇒Improved continuity compliance of anisotropic vortons leads to largely reduced parasitic pressure fluctuations

Computational Efficiency





jet mixer test case:

a). using recyling method (1100k cells)
b). using inflow generator (670k cells, ≈-40%)

(Identical resolution of the domain downstream of nozzle in both cases.)

Comparison

- recycling case ≈ 49700 sec. wallclock time / sec. simulation time
- rnflow generator: ≈ 26000 sec. wallclock time / sec. simulation time

⇒≈ +91% wallclock time by recycling method
⇒inflow generator sufficiently fast

Summary

- The Structure Based Method is a relatively simple and elegant method for synthesis of turbulent fields.
- Implementations so far were restricted to isotropic length scales and divergent velocity fields.
- The most important challenge is the combination of anisotropy with the divergence-free constraint.
- Formulation of anisotropic divergence-free vortons has been derived and implemented.

Outlook

- An extensive validation and comparison with other methods for turbulence generation is currently conducted.
- Extension for compressible flows and scalar fluctuations are planned.

Thank you for your attention!

Acknowledgements

- The work is supported by the Deutsche Forschungsgemeinschaft (DFG, grant KO 3394/7-1 and grant INST 264/113-1 FUGG)
- computational resources were granted by the North-German Supercomputing Alliance (HLRN)

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