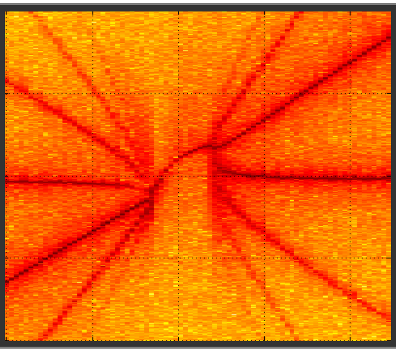
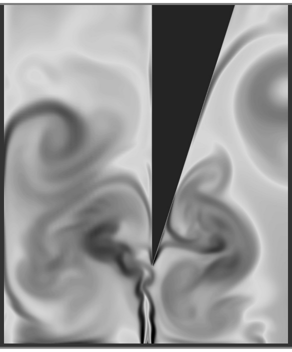
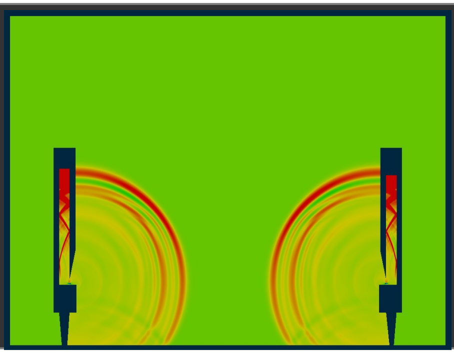
Navier-Stokes meets Synchronization

Numerical Simulations of Aeroacoustical Coupled Organ Pipes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Outline

- Synchronization in a nutshell
- Synchronization experiments with organ pipes
- Numerical simulations (CFD/CAA) of two aeroacoustically coupled organ pipes

Huygens 1665: Synchronization, 'an odd kind of synpathy'

Ability of special oscillatory systems to adjust their rhythms via interaction

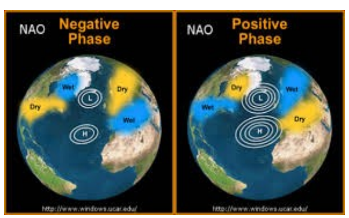
Systems are able to balance excitation and dissipation of energy intrinsically

→ Self-sustained Oscillators

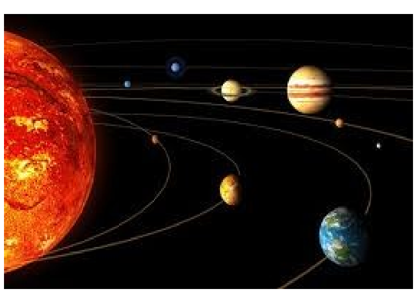
10^{-6}

10^{0}

10^{6}

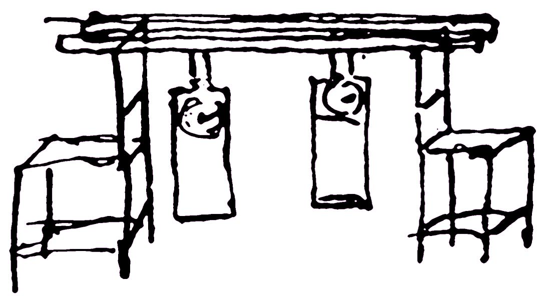


10^{12}



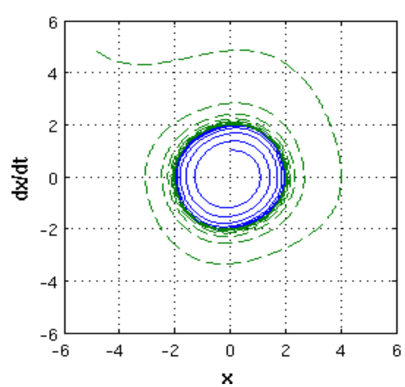
dimension [m]

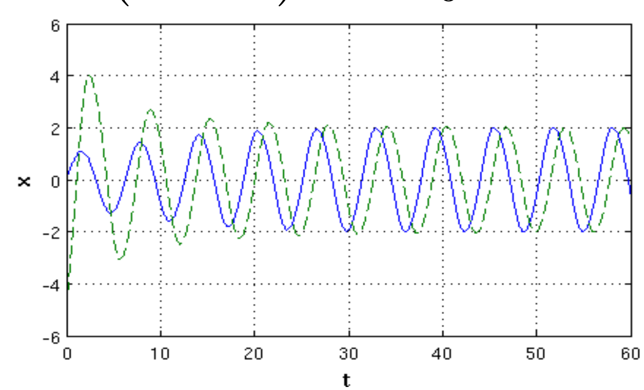
Huygens:



Lord Rayleigh: $\ddot{u} + \kappa\dot{u} + \kappa'\dot{u}^3 + n^2 u = 0$

Van der Pol Oscillator: $\ddot{x} - \mu\left(1 - x^2\right)\dot{x} + \omega_0^2 x = 0$





Phenomenology of Synchronization

Self-sustained Oscillator + driving external force (e.g. second Oscillator)

Example: $$\ddot{x} - \mu\left(1 - x^2\right)\dot{x} + \omega_0^2 x = \varepsilon\cos(\omega_e t + \phi_e)$$

ε : **Amplitude of driving periodic external force**

ω_e : **Frequency of the driving System**

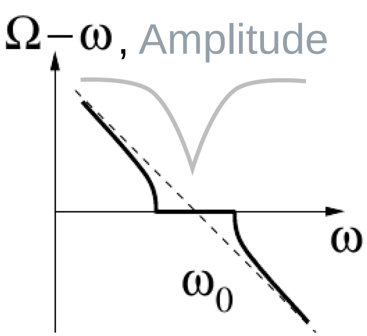
Ω, ω : **Observed Frequency of the driven and driving system**

ϕ_e : **Phase of the driving Periodic external force**

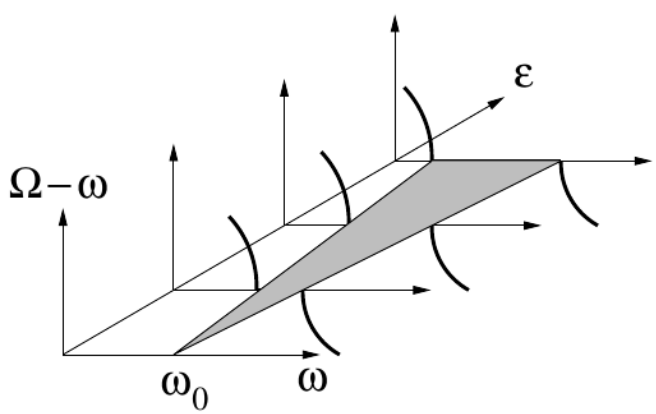
Mode locking: $\Omega - \omega = 0$

Phase locking: $\phi(t) + \phi_e(t) = const.$

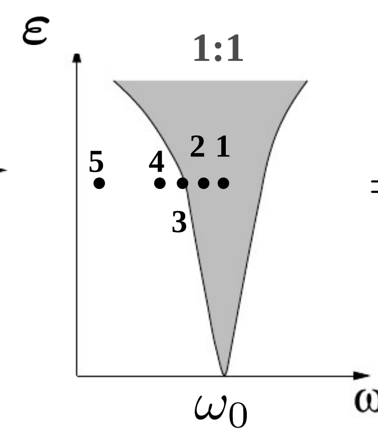
Synchronization plateau



Coupling strength

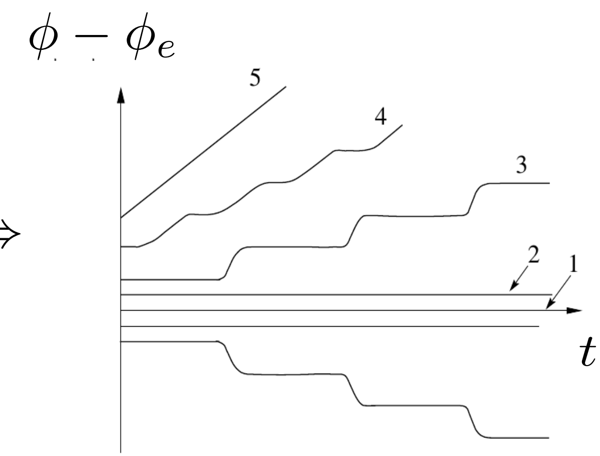


Arnold tongue



$\Rightarrow$

Phase relation

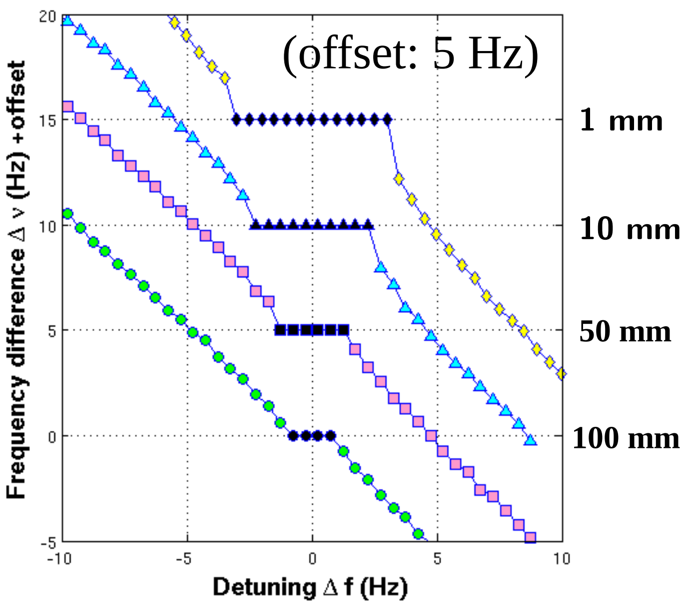


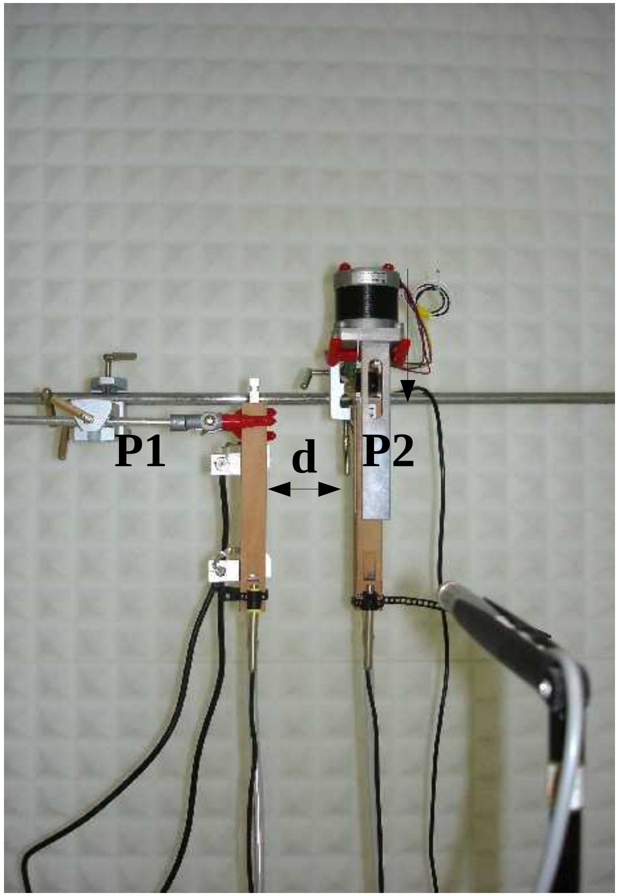
[Pictures: Pikovsky,Rosenblum, Kurths; **Synchronization – A universal concept of nonlinear sciences**, Cambridge Press 2001]

Synchronization of Two Organ Pipes

Measurements, Bergweiler, PhD-Thesis, 2006
Publication: Abel, Bergweiler, Gerhard-Multhaupt, JASA 2006,
Analysis: Fischer, Diploma, 2012, Coupling functions, Fischer, PhD-Thesis, 2014

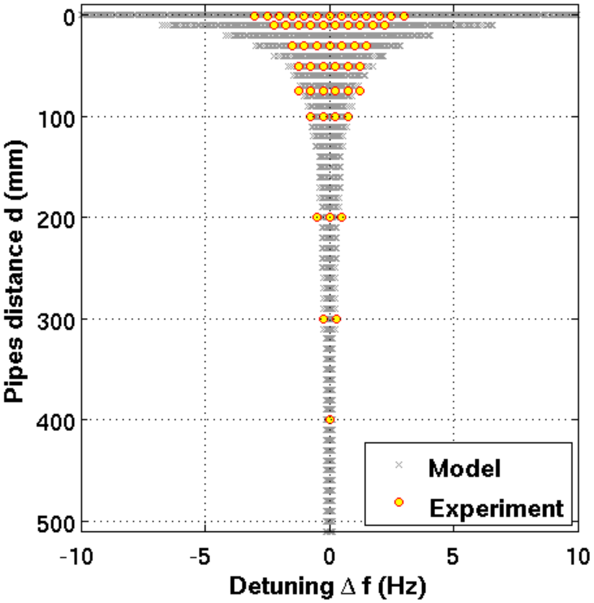
Synchronization plateaus





[Source: Abel, Bergweiler, 2005]

Arnold tongue



Parameter space: $(\Delta f, d)$

(Detuning, Pipe distance)

Synchronization of Two Organ Pipes

Measurements, Bergweiler 2006

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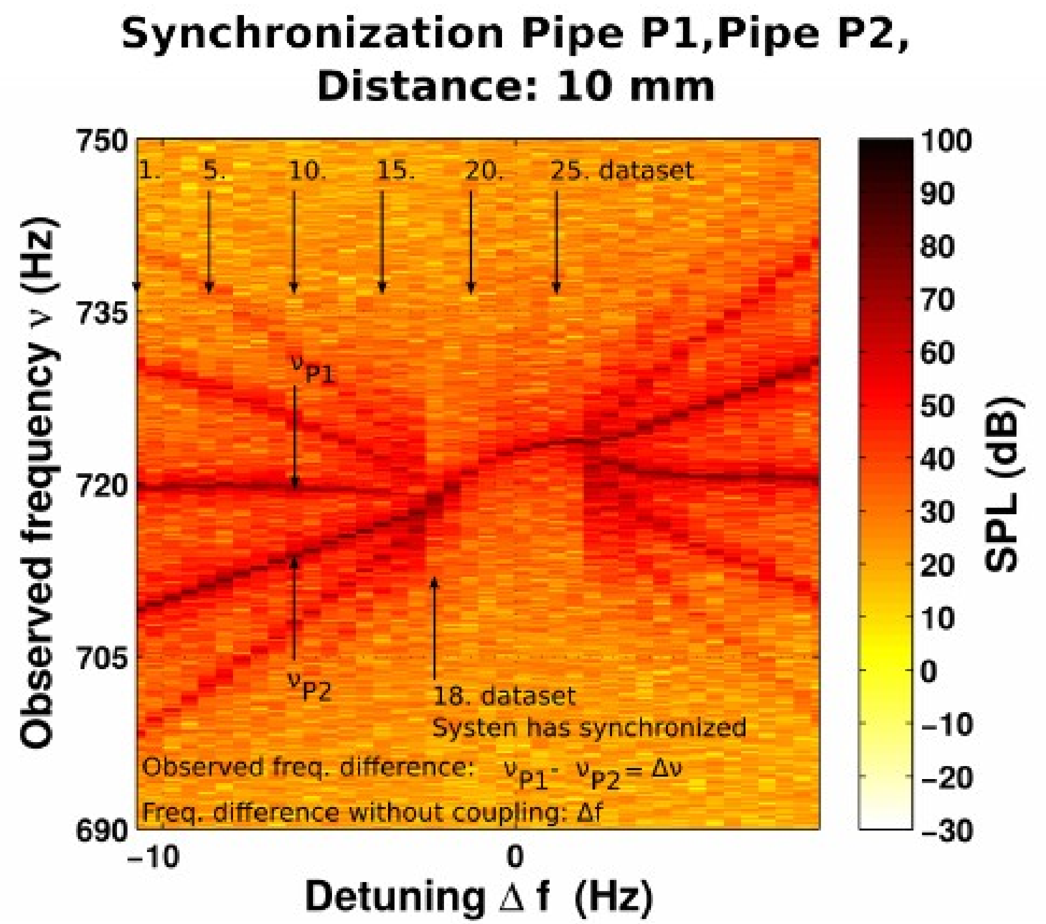
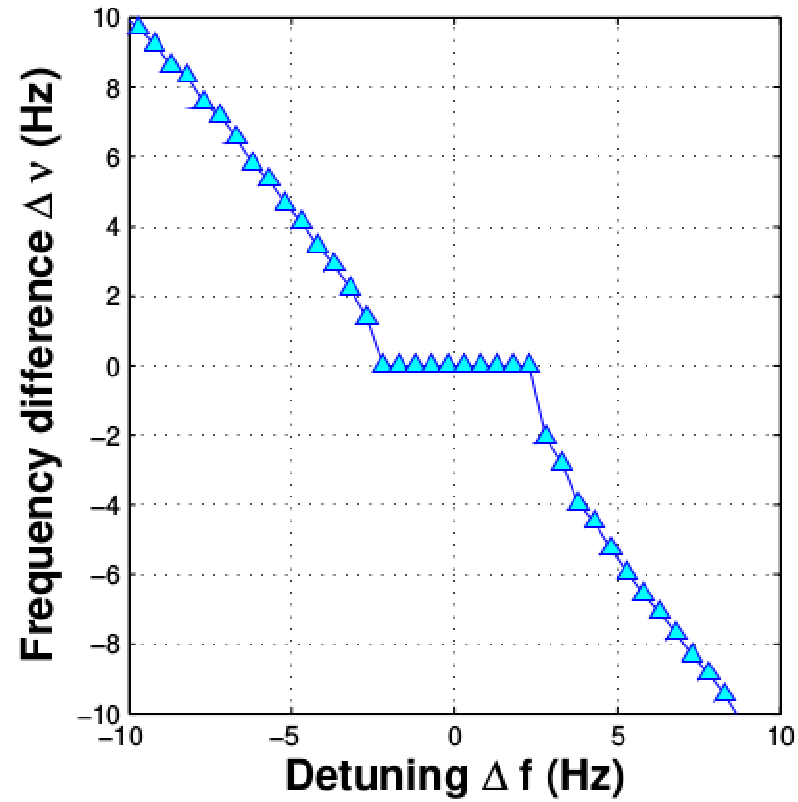


Synchronization plateau

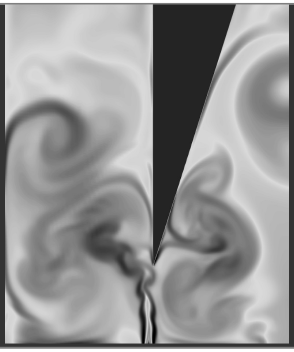


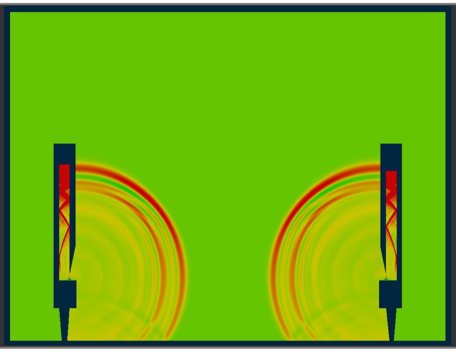
1. 5. 10. 15. 18. 20. 25.

Fluid Mechanical Sketch of the Problem

- Sound generation, sound propagation
- Turbulent jets
- Coupling wind field $\Longleftrightarrow$ acoustical field
- Sound propagation in wave guide and free space

- **inhomogenous, compressible**
- **turbulent**
- **instationary**
- **linear, nonlinear**





$\Longrightarrow$ **Compressible Navier-Stokes Equations (NSE)**

Numerical Implementation

Methods:
CFD/CAA

Preliminaries:	**Constituitive Equations (compressible NSE)** **Fluid mechanical characteristic numbers** **Kolmogorov-Scales, Grid size,** **Hardware, Software (256 CPU's, OpenFOAM)**
Pre-Processing:	**Mesh (Computational grid)** **Thermo-physical properties** **BC's, IC's** **Discretization** **Turbulence model (kEqn)** **Solver (rhoPimpleFoam)** **I/O Parameter, Simulation time** **Probe points, Samples**
Processing:	**Parallelization, Start** **Stability criteria** **Control in run time**
Post-Processing:	**Visualization** **Analysis**

The Constitutive Equations

6 Field varibles $(\rho, \boldsymbol{v}, p, T)$ velocity, density, pressure, temperature

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

Momentum balance

- **Compressible Navier-Stokes Equations** $(\nabla \cdot \boldsymbol{v} \neq 0)$

$$\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla) \boldsymbol{v} \right) + \boldsymbol{v} \frac{\partial \rho}{\partial t} = -\nabla p + (\lambda + \eta) \nabla (\nabla \cdot \boldsymbol{v}) + \eta \Delta \boldsymbol{v} + \rho \cdot \boldsymbol{g}$$

Energy equation

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho E \boldsymbol{v}) = -\nabla (p \boldsymbol{v}) + \nabla (\boldsymbol{\tau} \boldsymbol{v}) - \nabla \boldsymbol{q}$$

Fourier's law

$$\boldsymbol{q} = -\kappa \nabla T$$

The Constitutive Equations

$$\boldsymbol{U}_t + \boldsymbol{I}_x + \boldsymbol{J}_y + \boldsymbol{K}_z = 0$$

$$\boldsymbol{I} = \boldsymbol{I}^{inv} - \boldsymbol{I}^{vis}, \quad \boldsymbol{J} = \boldsymbol{J}^{inv} - \boldsymbol{J}^{vis}, \quad \boldsymbol{K} = \boldsymbol{K}^{inv} - \boldsymbol{K}^{vis}$$

$$\boldsymbol{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{pmatrix}, \quad \boldsymbol{I}^{inv} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho u w \\ \rho u E + u p \end{pmatrix}, \quad \boldsymbol{I}^{vis} = \begin{pmatrix} 0 \\ \sigma_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\sigma_{xx} + v\tau_{xy} + w\tau_{xz} - q_x \end{pmatrix}$$

$$\boldsymbol{J}^{inv} = \begin{pmatrix} \rho v \\ \rho v u \\ \rho v^2 + p \\ \rho v w \\ \rho v E + v p \end{pmatrix}, \quad \boldsymbol{J}^{vis} = \begin{pmatrix} 0 \\ \tau_{yx} \\ \sigma_{yy} \\ \tau_{yz} \\ u\tau_{yx} + v\sigma_{yy} + w\tau_{yz} - q_y \end{pmatrix}$$

$$\boldsymbol{K}^{inv} = \begin{pmatrix} \rho w \\ \rho w u \\ \rho w v \\ \rho w^2 + p \\ \rho w E + w p \end{pmatrix}, \quad \boldsymbol{K}^{vis} = \begin{pmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \sigma_{zz} \\ u\tau_{zx} + v\tau_{zy} + w\sigma_{zz} - q_z \end{pmatrix}$$

Fluid Mechanical Characteristic Numbers

Mach

$$Ma = \frac{v}{c_0} \approx \frac{18\,m/s}{340\,m/s} = 0.053$$

Strouhal

$$Sr = \frac{f \cdot l}{v} \approx \frac{700\,Hz \cdot 5.5 \cdot 10^{-3}\,m}{18\,m/s} = 0.21$$

Prandtl

$$Pr = \frac{\nu}{a} \approx \frac{15 \cdot 10^{-6}\,m^2/s}{20 \cdot 10^{-6}\,m^2/s} = 0.75$$

Reynolds

$$Re = \frac{v \cdot l}{\nu} \approx \frac{18\,m/s \cdot 5.5 \cdot 10^{-3}\,m}{15 \cdot 10^{-6}\,m^2/s} = 6600$$

Péclet

$$Pe = \frac{v \cdot l}{a} = Re \cdot Pr \approx 4950$$

Kolmogorov-Scales, Grid Size

Kolmogorov-length

$$\eta_k = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

Dissipation rate

$$\epsilon \sim \frac{v^3}{l}$$

Kolmogorov-time scale

$$\tau_k = \left(\frac{\nu}{\epsilon}\right)^{1/2}$$

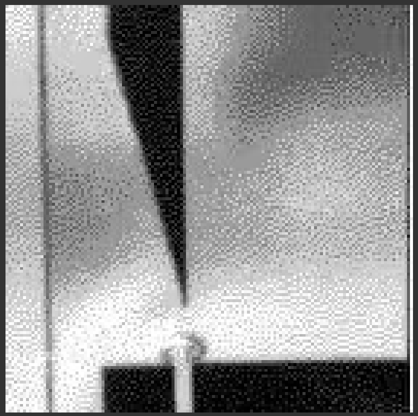
Characteristic velocity (speed of sound) $c_0 \approx 340\, m/s$

$$\Rightarrow \qquad \eta_k \approx 1 \cdot 10^{-6}\, m \qquad \tau_{k,c_0} \approx 1.6 \cdot 10^{-8}\, s$$

3D DNS Setup: $L \times W \times H \quad 400\, mm \times 200\, mm \times 200\, mm$

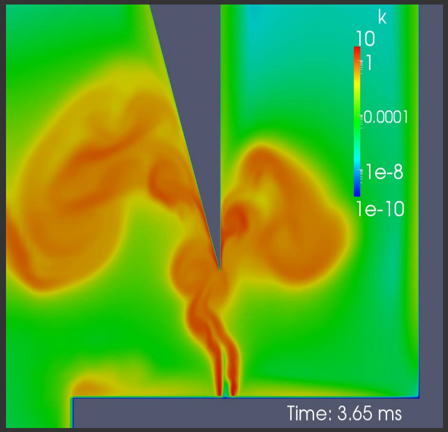
$\Rightarrow 1.6 \cdot 10^{16} \times 1.6 \cdot 10^{8} s^{-1} = 2.56 \cdot 10^{24}$ Ops/s **not realizable**

Pseudo-3D LES Setup: $\Rightarrow 1.25 \cdot 10^{6} \times 1 \cdot 10^{7}\, s^{-1} = 1.25 \cdot 10^{13}$ Ops/s + LES-Modell



Oscillating jet
in a real organ pipe

M. P. Verge, B. Fabre,
A.Hirschberg,
A.P.J. Wijnands,1994



Numerical simulation
of the generator region

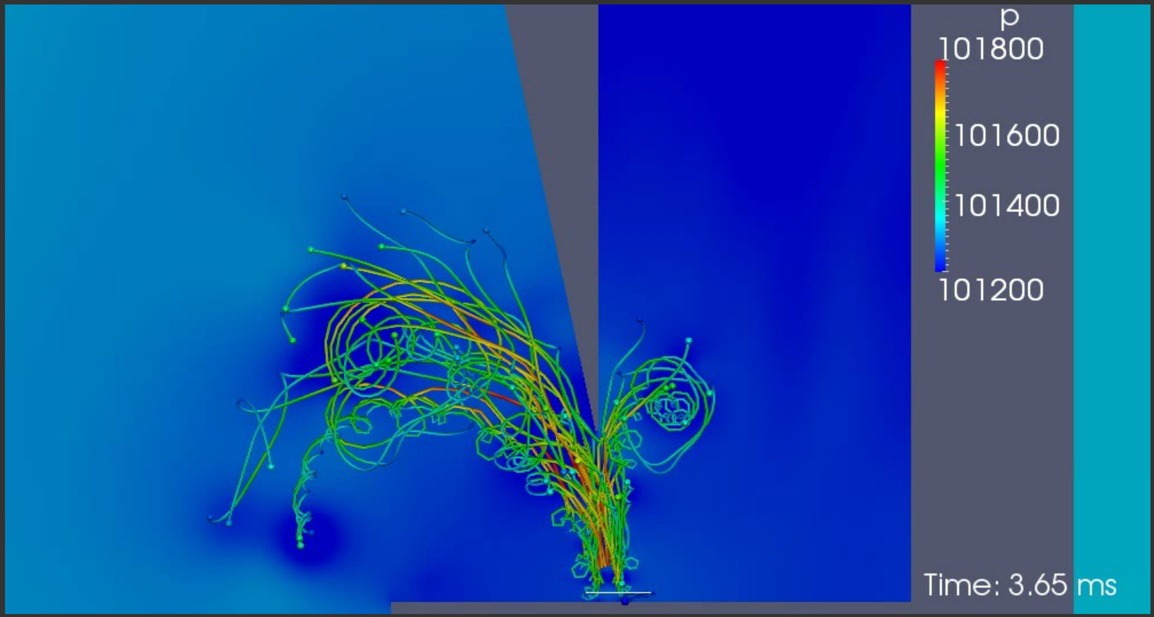
Color coded is the turbulent kinetic energy k, log-scaled

J.L.Fischer 2013

Particle Tracking at the Organ Pipes Jet

Path lines: Color coded is the magnitude velocity UMag
Background: Color coded is the pressure p

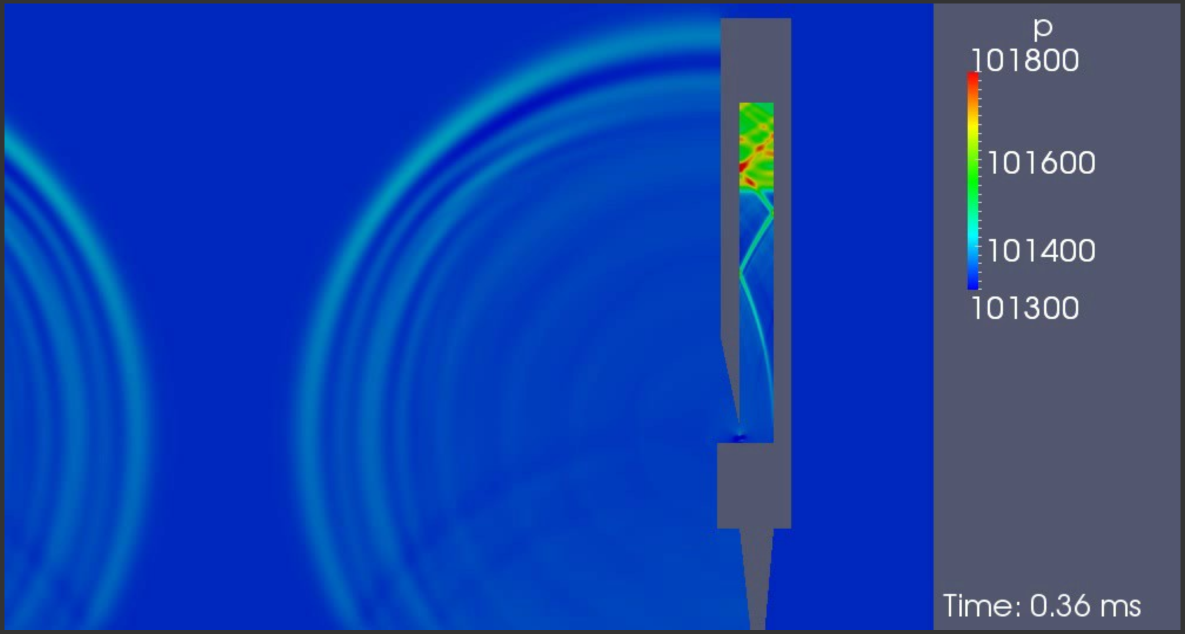
J.L.Fischer, 2015



Initial sound wave propagating in an organ pipe. Radiation of spherical sound waves.

Detail of a numerical simulation of the interaction of two organ pipes
Color coded is the pressure p

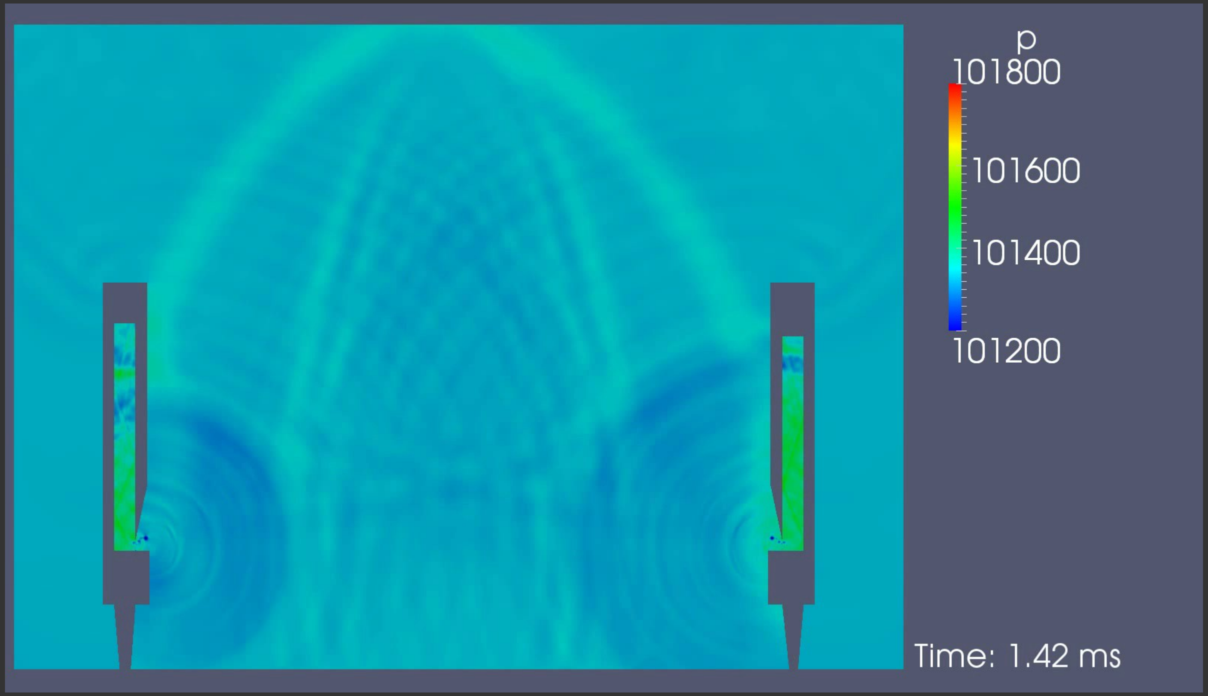
J.L.Fischer 2016



Numerical simulation of synchronization of the pipe-pipe system

Color coded is the pressue p

J.L.Fischer, 2016



P1 **P2**

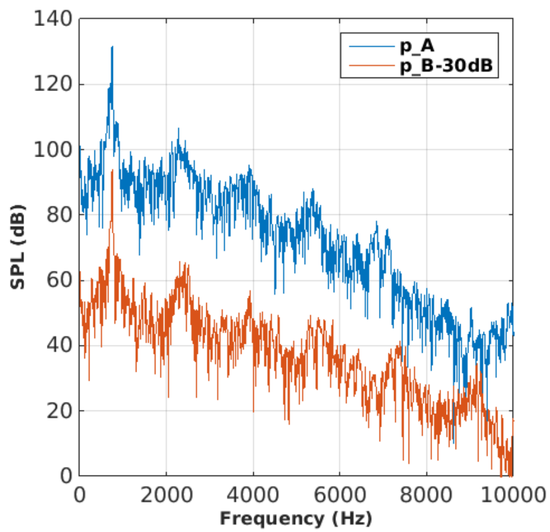
SPL-Spectum of the Coupled Pipe-Pipe System

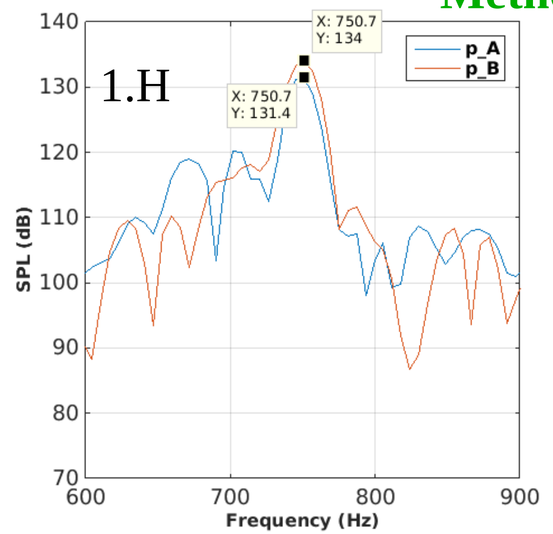
Autonomous frequencies: $f_{P1} = 732\,Hz,\ \ f_{P2} = 756\,Hz$

Methods: Fourier, SPL

Signal length:
t=86.60ms
(8860 pts)

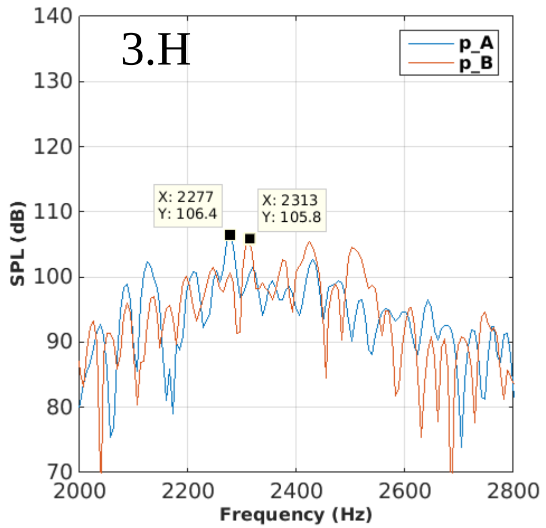
Sampling frequency:
1e5 Hz

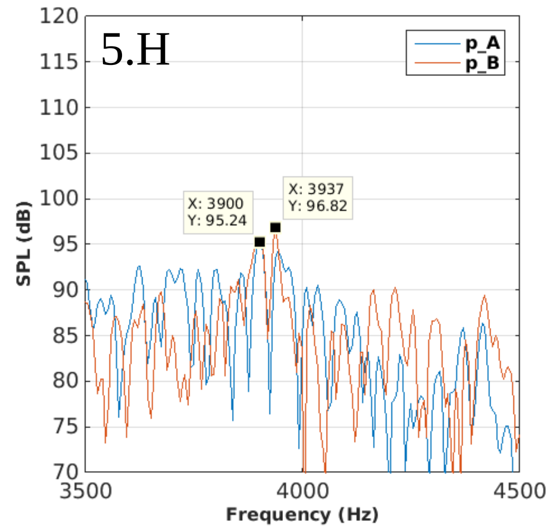




1.H: 750.7 Hz
(100%)

3.H: 2252.1 Hz
(101% / 102%)



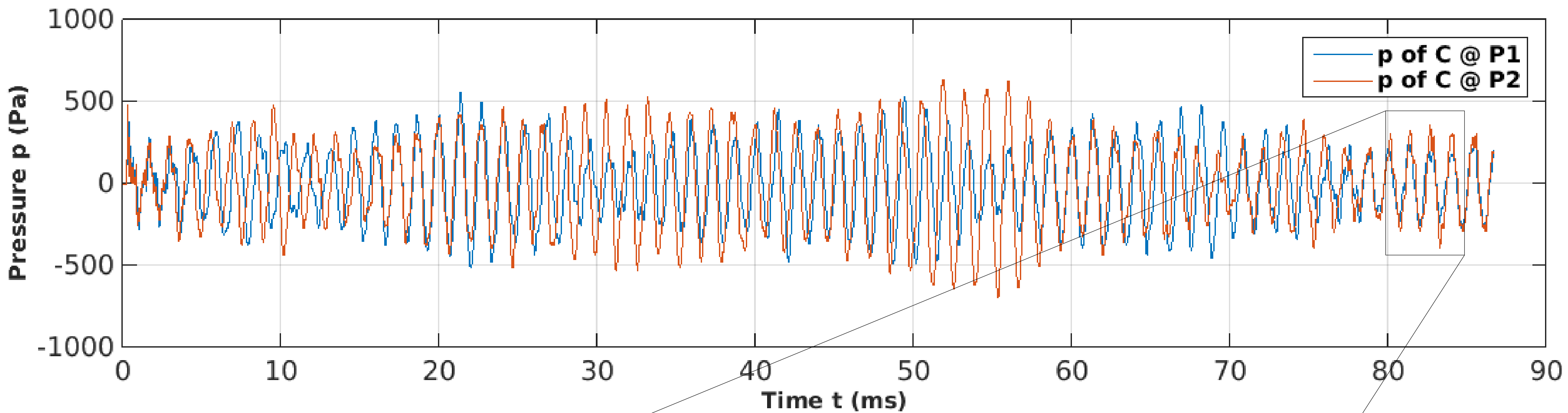


5.H: 3753.5 Hz
(104% / 105%)

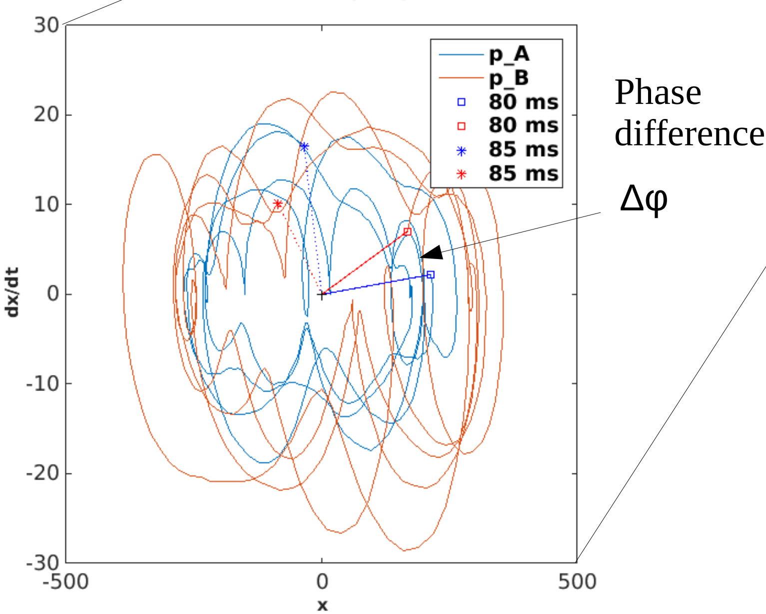
Numerical Simulation of Synchronization of the Pipe-Pipe System $f_{P1} = 732\,Hz, \; f_{P2} = 756\,Hz$

Pressure signals at probe points C in the resonators of organ pipes P1 and P2

Methods: Sampling

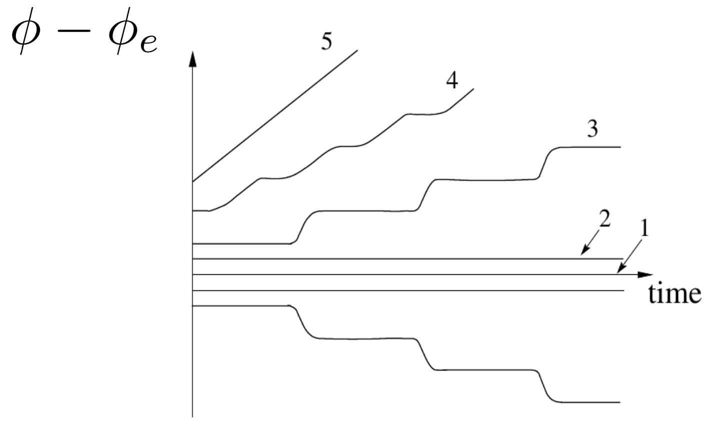


Methods:
Phase angle reconstruction in phase space



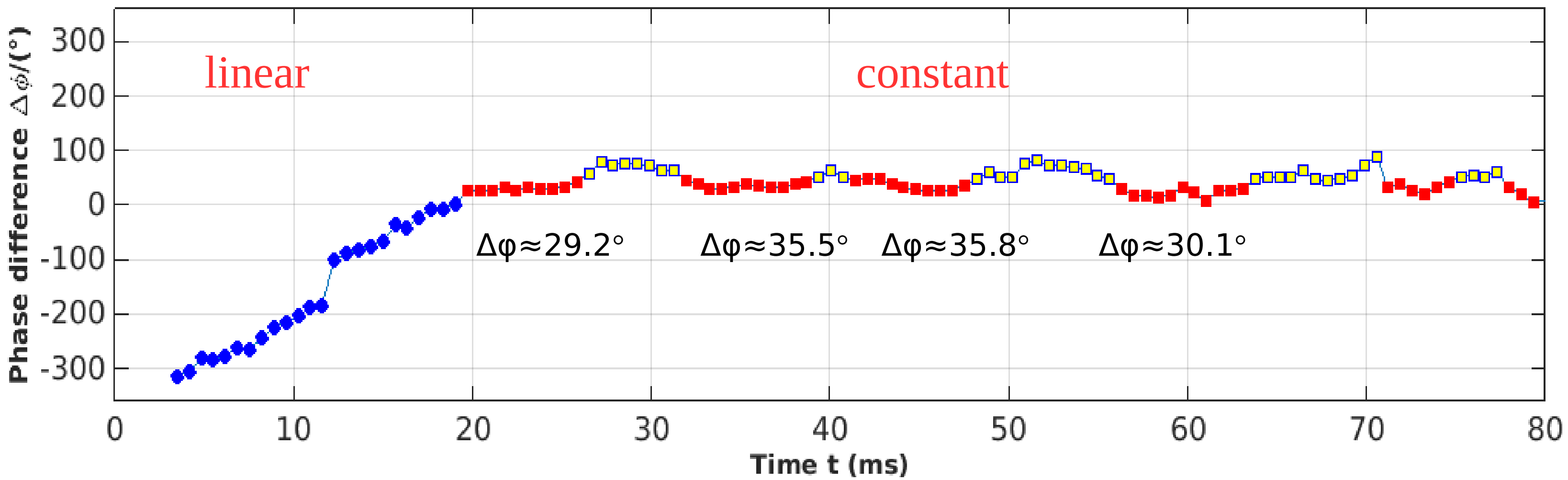
Numerical Simulation of Synchronization of the Pipe-Pipe System $f_{P1} = 732\,Hz,\; f_{P2} = 756\,Hz$

Theory:



Numerical simulation:

Phase difference Δφ at signals zero-crossing of the coupled pipe-pipe system



Summary

- It is in principle possible to describe and investigate the phenomenon of synchronization of two aeroacoustical coupled organ pipes by solving the compressible NSE numerically

- **Exciting open questions:** Nonlinearities inside the organ pipe
  - Turbulent jet
  - Primary vortex
  - Friction at the resonator's inner walls
  - Function of the labium
  - Interplay

- **Outlook:** Phase analyses of the relevant fluid dynamical elements:
  jet, primary vortex, resonator's roughness, acoustical dipole at the labium,...

Thanks for Your Attention!