4^{th} Northern germany OpenFoam User meetiNg 2016

3D Simulation of an Argon magnetoplasmadynamic thruster with coaxial induced magnetic field

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DAAD Deutscher Akademischer Austauschdienst German Academic Exchange Service

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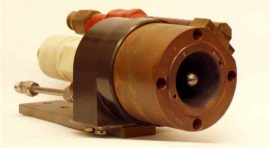
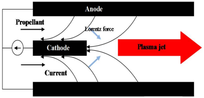
What is Magnetoplasmadynamic thruster?

Definition

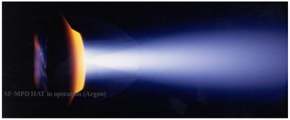
A Magnetoplasmadynamic thruster (MPDT) is a form of electrically powered spacecraft propulsion which uses the Lorentz force to generate thrust.



NASA[1]
200-Kilowatt MPDT



Schematic view of a self-field MPDT



MPDT in operation[2]

[1] National Aeronautics and Space Administration

[2] NASA Facts, Glenn Research Center

Motivation

Physical problems

- Real gas effects
- Thermal nonequilibrium
- Problem of friction
- Difficulty to seperate flow and discharge

Numerical difficulties

- The necessary coupling of partial differential equations systems (Elliptical and hyperbolic)
- Nonlinearities

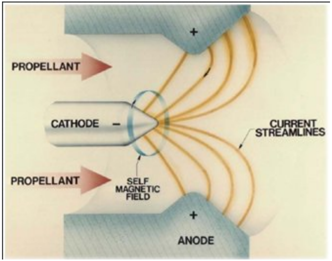
The goals of the present work

- Calculate a plasma flow in the self-field MPDT by using a Central-Upwind scheme
- Obtain insight into the physics of thrust production and Energy dissipation

Physical model

The model ...

we present is dedicated to the description of an electrically conducting but electrically neutral fluid.



Overview of MPDT operation [3]

We assumed that:

- The propellant gas (Argon) is injected into the discharge chamber as fully-ionized fluid
- The plasma flow is in a state of thermal equilibrium $T \approx T_e \approx T_i$
- Electrical sheat, Hall effect and radiation processes are neglected.

[3] NASA Facts, Glenn Research Center

Governing equations

For compressible MHD flow:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{U} = 0$$

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot [\rho \mathbf{U}\mathbf{U} + (p + \frac{B^2}{2\mu_0})\mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{\mu_0}] = \nabla \cdot \tau_{visc}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B} - \mathbf{B}\mathbf{U}) = -\frac{1}{\mu_0 \sigma} \Delta \mathbf{B}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + p + \frac{B^2}{2\mu_0})\mathbf{U} - \frac{\mathbf{B}\mathbf{B}}{\mu_0}] = \nabla \cdot [k_{th} \nabla T - (\frac{\eta \mathbf{J} \times \mathbf{B}}{\mu_0})]$$

ρ, $\mathbf{U}$ and $\mathbf{B}$: *are the average density of all species, the velocity and magnetic field vectors*

$\rho E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho U^2 + \frac{B^2}{2\mu_0}$ *is the total energy density of the plasma*

Permeability of the free space μ_0, Curent density J, Electrical conductivity σ, Thermal conductivity coefficient k_{th}

Governing equations

Electrical conductivity ...

is deducted from the Spitzer-Harm formulation [a]:

[a] Spitzer, L. and Härm, R.: *Transport phenomena in a completely ionized gas*. Phys. Rev., 89, 977 (1953)

$$\sigma = 1.53 \times 10^{-2} \frac{T^{\frac{3}{2}}}{ln\Lambda} \qquad ln\Lambda = ln[\frac{12\sqrt{2}\pi(k_B \epsilon_0 T)^{\frac{3}{2}}}{q^3 n^{\frac{1}{2}}}]$$

The divergence Cleaning method ...

is coupled with the induction equation to ensure that $div(B) = 0$ [a]

[a] Dedner, A., Kemm, F., Munz, C.D., Schnitzer, T., and Wesenberg, M.: *Hyperbolic Divergence Cleaning for the MHD Equations*. J. Comput. Phys., 175, 645-673 (2002)

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{UB} - \mathbf{BU}) + \nabla \psi = -\frac{1}{\mu_0 \sigma} \Delta \mathbf{B} \qquad c_h = \frac{CFL}{\Delta t \times max(\frac{1}{h})}$$

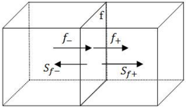
$$\frac{\partial \psi}{\partial t} + c_h^2 \nabla \cdot \mathbf{B} = -\frac{c_h^2}{c_d^2} \psi \qquad c_d = \sqrt{-\Delta t \frac{c_h^2}{ln(C_r)}}$$

Computational Method

Summerized convective and diffusive fluxes [4] with divergence cleaning terms:

$$\sum_f \phi_f W_f = \sum_f [\alpha\phi_{f+}(W_{f+} + \kappa_{f+}) + (1-\alpha)\phi_{f-}(W_{f-} + \kappa_{f-}) + \varpi_f(W_{f-} - W_{f+}) + \frac{1}{2}(\Phi_{f-} + \Phi_{f+})]$$

$$\mathbf{W}_f = \begin{pmatrix} \rho \\ \rho U_x \\ \rho U_y \\ \rho U_z \\ B_x \\ B_y \\ B_z \\ \rho E + P_{over} \\ 0 \end{pmatrix}, \kappa_f = \begin{pmatrix} 0 \\ S_x P_{over} \\ S_y P_{over} \\ S_z P_{over} \\ -b_f U_x \\ -b_f U_y \\ -b_f U_z \\ -b_f(\mathbf{U}\cdot\mathbf{B}) \\ 0 \end{pmatrix}, \varpi_f = \begin{pmatrix} 0 \\ \omega_f \\ \omega_f \\ \omega_f \\ 0 \\ 0 \\ 0 \\ \omega_f \\ 0 \end{pmatrix}, \Phi_f = \begin{pmatrix} 0 \\ -B_x b_f \\ -B_y b_f \\ -B_z b_f \\ S_x\psi \\ S_y\psi \\ S_z\psi \\ 0 \\ c_h^2 b_f \end{pmatrix}$$



Central-upwind interpolation schema

[4] Kurganov, A., Noelle, S., Petrova, G.: *Semi-discrete central-upwind schemes for hyperbolic conservation laws and Hamilton-Jacobi equations*. SIAM J. Sci. Comput., 23, 707-740 (2001).

Central Method Coefficients

Convective Terms

$$\alpha = \begin{cases} \frac{1}{2} & \text{for the KT method} \\ \frac{\Psi_{f+}}{\Psi_{f+}+\Psi_{f-}} & \text{for the KNP method} \end{cases}$$

Diffusive Terms

$$\omega_f = \begin{cases} \alpha max(\Psi_{f+}, \Psi_{f-}) & \text{for the KT method} \\ \alpha(1-\alpha)(\Psi_{f+}+\Psi_{f-}) & \text{for the KNP method} \end{cases}$$

Central Method fluxes [5]

$$\Psi_{f+} = max(c_f|S_f| + \phi_{f+}, c_f|S_f| + \phi_{f-}, 0)$$

$$\Psi_{f-} = min(c_f|S_f| - \phi_{f+}, c_f|S_f| - \phi_{f-}, 0)$$

$$c_f = min(c_+, c_-)$$

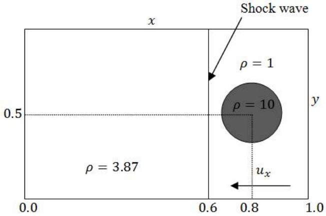
Effective speed of sound

$$c_\pm = (\frac{1}{2}[a_\pm^2 + \frac{B_\pm^2}{\mu_0 \rho_\pm} + \sqrt{(a_\pm^2 + \frac{B_\pm^2}{\mu_0 \rho_\pm})^2 - 4a_\pm^2 \frac{B_{n,\pm}^2}{\mu_0 \rho_\pm}}])^{\frac{1}{2}}$$

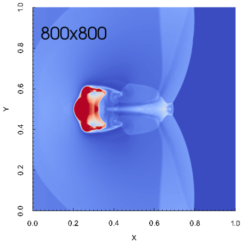
[5] Greenshields, C.J., Weller, H.G., Gasparini, L., and Reese, J.M.: *Implementation of semi-discrete, non-staggered central schemes in a collocated, polyhedral, finite volume framework, for high-speed viscous flows*. Int. J. Numer. Meth. Fluids, 63, 1-21 (2010).

The Shock-cloud interaction problem

$$(\rho, u_x, u_y, u_z, p, B_x, B_y, B_z) = \begin{cases} (3.86, 0, 0, 0, 167.34, 0, 2.18, -2.18) & \text{if } x < 0.6 \\ (1, -11.25, 0, 0, 1, 0, 0.56, -0.56) & \text{if } x \geq 0.6 \end{cases}$$



(a) Initial conditions for density and the geometry used for the cloud-shock interaction test case

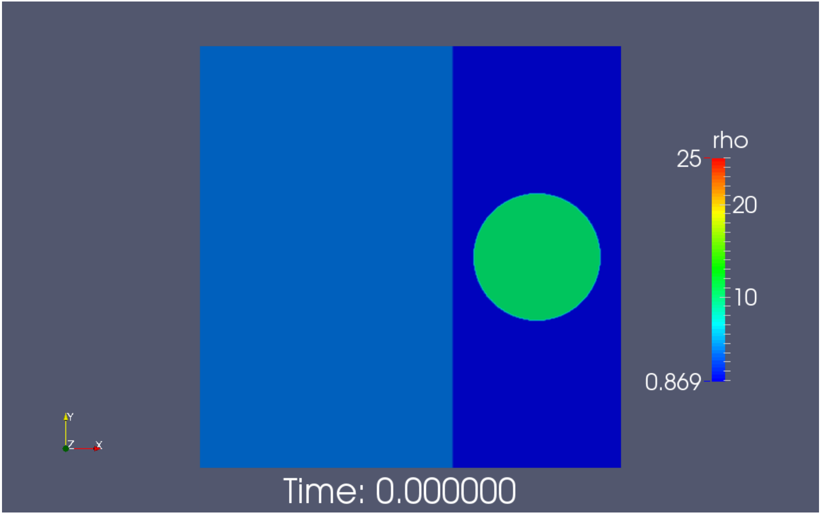


(b) Density distribution

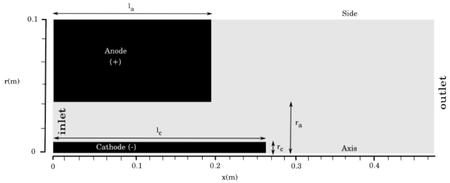
Initial conditions for density and the geometry used for the cloud-shock interaction test case [6] and density distribution on the $N = 800 \times 800$ grid at $t = 0.06$ s. Density in (kg/m^3)

[6] Xisto, C.M., Pascoa, J.C., Oliviera, P.J.: *A pressure-based high resolution numerical method for resistive MHD*. J. Comput. phys., 275, 323-345 (2014).

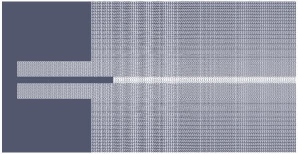
Density



Villani-H thruster: Case set-up



Geometry of the Villani-H thruster [7]



Mesh grid of the Villani-H thruster with about 1.25 millions cells

- Input of the code:
  Discharge Current I (A)
  Temperature T (K)
  The propellant mass flow rate $\dot{m}$ (kg/s)

$$B_0 = \frac{\mu_0 I}{2\pi r}$$

GroovyBC

$$p = \rho R T$$

[7] K. Sankaran, 2005

Boundary conditions

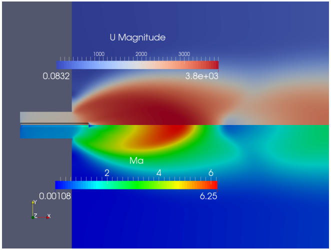
Variables	Inlet	Electrodes	Insulated walls	Outlet
U (m/s)	$\dot{m} = 6.0(g/s)$	slip/non slip	non-slip	ZG
T (eV)	fixedValue 1.0	ZG	ZG	ZG
p (Pa)	ZG	ZG	ZG	WT
B (T)	TDBC	Conducting walls	fixedValue $(0,0,0)$	fixedValue $(0,0,0)$

Boundary conditions
of the MPDT simulations

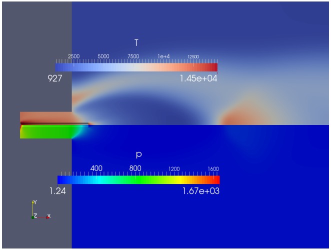
- ZG: Zero gradient
- WT: Wave transmissive
- TDBC: Time depending boundary condition

$$B = \begin{cases} 0, & \text{if } t < t_1. \\ B_0 \frac{t-t_1}{t_2-t_1}, & \text{if } t_1 < t < t_2. \\ B_0, & \text{if } t > t_2 \end{cases}$$

Results



(c) Initial conditions for density and the geometry used for the cloud-shock interaction test case



(d) Density distribution

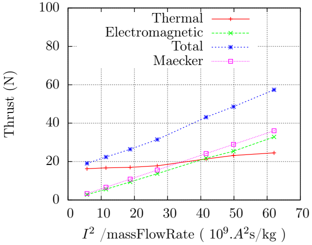
Velocity, Mach number distribution (left), Temperature and pressure (right) with $\frac{I^2}{\dot{m}} = 05 \times 10^9 \cdot \frac{A^2 s}{kg}$, $l_c = 0.264\ m$, $r_c = 0.0095\ m$, $r_a = 0.051\ m$ and $\frac{l_a}{r_a} = 04$ for MPDT02. Units: Velocity (m/s), Temperature (K), Pressure (pa) and Density (kg/m^3)

Geometric scaling analysis

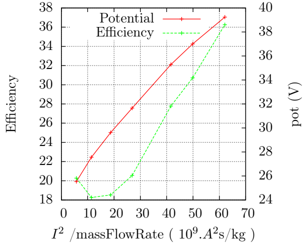
MPDT geometries and $\frac{I^2}{\dot{m}}$ values for the numerical parameter study on the HLRN. r_a, l_a and l_c denote anode radius, anode length and cathode length respectively.

Case geometry	$\frac{l_a}{r_a}$	$\frac{I^2}{\dot{m}}[10^9 \cdot \frac{A^2 s}{kg}]$	$l_c[m]$
MPDT01	1	5.7	0.132
$r_c = 0.0095[m]$	2	11.2	0.264
$r_a = 0.025[m]$	3	18.4	
	4	25.6	
	5	48.4	
		60.2	
MPDT02	1	5.7	0.132
$r_c = 0.0095[m]$	2	11.2	0.264
$r_a = 0.051[m]$	3	18.4	
	4	25.6	
	5	48.4	
		60.2	
MPDT03	1	5.7	0.132
$r_c = 0.0181[m]$	2	11.2	0.264
$r_a = 0.051[m]$	3	18.4	
	4	25.6	
	5	48.4	
		60.2	

Geometric scaling analysis



(e) Thrust

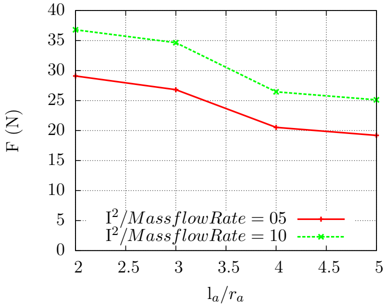


(f) Specific impulse and current voltage

Thrust, efficiency, plasma current voltage for MPDT01 with $\frac{l_a}{r_a} = 04$.

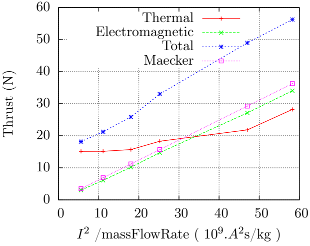
$\frac{I^2}{\dot{m}}[10^9 \cdot \frac{A^2 s}{kg}]$	$F_{EM}[N]$	$Maecker[N]$	$F_{thermal}[N]$	$F_{total}\ [N]$	$V_{plasma}[V]$	$Isp[s]$
60.2	32.86	36.06	24.54	57.4	39.238	975.13

Geometric scaling analysis

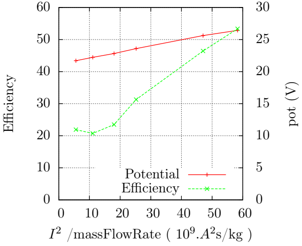


Thrust function of aspect ratio for MPDT01 for $\frac{I^2}{\dot{m}} = 05.7 \times 10^9 \cdot \frac{A^2 s}{kg}$ and $\frac{I^2}{\dot{m}} = 11.2 \times 10^9 \cdot \frac{A^2 s}{kg}$.

Geometric scaling analysis



(g) Thrust

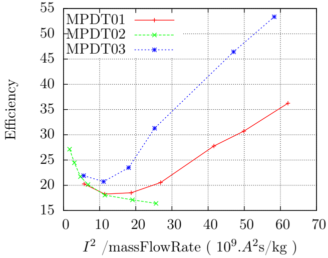


(h) Specific impulse and current voltage

Thrust, efficiency, plasma current voltage for MPDT03 with $\frac{l_a}{r_a} = 04$.

$\frac{I^2}{\dot{m}}[10^9 \cdot \frac{A^2 s}{kg}]$	$F_{EM}[N]$	$Maecker[N]$	$F_{thermal}[N]$	$F_{total}\ [N]$	$V_{plasma}[V]$	$Isp[s]$
60.2	34.04	36.24	28.20	56.24	26.425	955

Geometric scaling analysis



Thrust efficiency (%) for both MPDT01, MPDT02 and MPDT03 with $\frac{l_a}{r_a} = 04$.

$$\eta_{eff} = \frac{F_{total}^2}{2\dot{m}I(V + V_{sh})}$$

V_{sh}=20 V
(K. Sankaran, 2005)

Case geometry	η_{effmax}[%]	η_{effmin}[%]
$MPDT01$	36.25	20.27
$MPDT02$	27.14	16.41
$MPDT03$	53.35	21.9

Conclusion

- The central-upwind schemes have been succefully extended to MHD equations.
- The solver has demonstrated capability to compute resistive plasma flows in simple geometries

What remain to be done?

- improve the physical model (by adding Hall effect, real gas effect, considering multiple fluid plasma flow, ...) in order to achieve a predominantly electromagnetic acceleration mode for all thruster configuration in more realistic scenario.

Thank You for your Attention!

$\mathbf{j} \times \mathbf{B}$: *Lorentz force of a volume element*

$$\mathbf{j} \times \mathbf{B} = \frac{1}{\mu}(\nabla \times \mathbf{B}) \times \mathbf{B} = \underbrace{\frac{1}{\mu}(\mathbf{B} \cdot \nabla)\mathbf{B}}_{\textit{mag. Diffusion}} - \underbrace{\nabla\left(\frac{B^2}{2\mu}\right)}_{\textit{mag. Pressure}}$$

Maecker (1955) introduces an analytical expression of the electromagnetic thrust for coaxial self-field MPD thrusters, based on continuum plasma and ideal MPD approximation.

$$F_{Maecker} = \frac{\mu_0}{4\pi} I^2 (ln \frac{r_a}{r_c} + A)$$

where A is a dimensionless constant between 0 and 1. In this study, we considered $A = 0$.

Density ρ, Permeability of the free space μ_0, Curent density **j**

The MHD shock tube problem: Magnetic shock wave

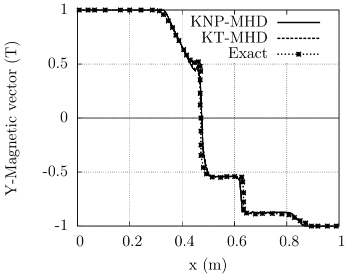
left	right
$\rho = 1\ kg/m^3$	$\rho = 0.125\ kg/m^3$
$p = 1\ Pa$	$p = 0.1\ Pa$
$U = (0, 0, 0)\ m/s$	$U = (0, 0, 0)\ m/s$
$B = (0.75, 0, 0)\ T$	$B = (0.75, -1, 0)\ T$

Initial conditions
of the problem [8]

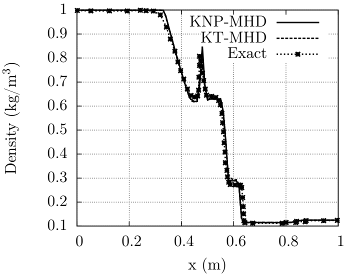
- Density jump
- Pressure jump
- Cross-sheared magnetic unsteadiness

[8] Sod, G.A.:*A survey of several finite difference methods for systems of nonlinear hyperbolic conservation laws*. J. Comput. phys., 27, 1-31 (1978)

The MHD shock tube problem: Magnetic shock wave



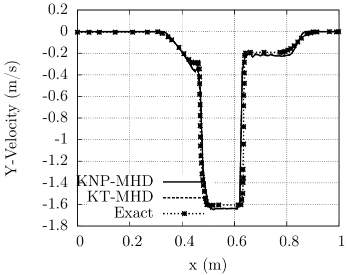
(i)



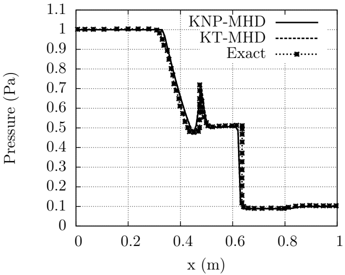
(j)

Comparaison of exact y-component of magnetic field (a)and density (b) profiles with numerical simulation results at $t = 0.1\ s$

The MHD shock tube problem: Magnetic shock wave



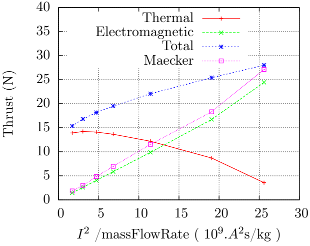
(k)



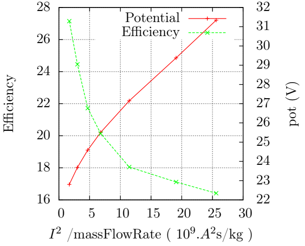
(l)

Comparaison of exact y-component of velocity (a) and pressure (b) profiles with numerical simulation results at $t = 0.1\ s$

Geometric scaling analysis



(m) Thrust



(n) Specific impulse and current voltage

Thrust, efficiency, plasma current voltage for MPDT02 with $\frac{l_a}{r_a} = 04$.

$\frac{I^2}{\dot{m}}[10^9 \cdot \frac{A^2 s}{kg}]$	$F_{EM}[N]$	$Maecker[N]$	$F_{thermal}[N]$	F_{total} $[N]$	$V_{plasma}[V]$	$Isp[s]$
25.2	24.43	27.2	3.57	28	31.34	475