Technische Universität Braunschweig

Adjoint thermal optimization

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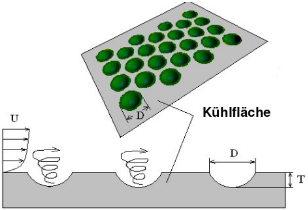
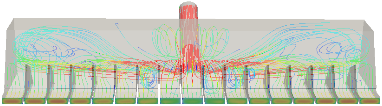
Applications

- Heat transfer on dimpled surfaces



- Uniformity at HVAC outlets

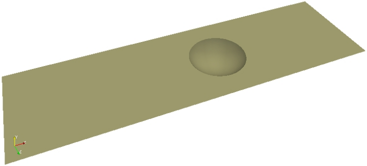


- ...

Joined work with *Johan Turnow, Uni Rostock*

Motivation

Optimization
Dimpled surfaces



- Dimpled geometries shows the best thermal-hydraulic performance (compared to ribs and fins)
- Thermal-hydraulic performance is

$$\frac{\text{heat exchange}}{\text{pressure loss increases}}$$ [1]

[1] Ligrani, Oliveira, Blaskovich T., Comparison of heat transfer augmentation techniques *AIAA Journal*,**41** (3), 337–361, (2003).

Formulation

- Let I be a specific cost function
- $\Omega \subset \mathbb{R}^N$ an admissible domain with boundary Γ
- Typically, the form has to satisfy a set of given constraints $\mathcal{R} = 0$ mostly defined as PDEs with state variables U.
- The form is parametrized by of set of design variables β
- We can formulate the problem by

$$\min_{\beta} / \max I(\mathbf{s}, \alpha) \quad \text{subject to} \quad \mathbf{r}(\mathbf{s}, \beta) = 0 \text{ on } \Omega$$

Governing equations

We start from the incompressible Navier-Stokes equations:

$$\begin{aligned} \partial_t(\rho \mathbf{u}) + (\mathbf{u} \cdot \nabla)\mathbf{u} &= -\nabla p + \nabla \cdot [2\nu D(\mathbf{u})] \\ \partial_t \rho + \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

with

- p pressure, $\mathbf{u} = (u_1, \ldots, u_3)^T$ velocity, T Temperature
- $D(\mathbf{u}) = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$ stress tensor
- ν kinematic viscosity

We equip the system with an thermal diffusion equation, i.e.

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \nabla(\alpha \cdot \nabla T)$$

with α thermal diffusivity.

Residual form of the N.-S. system

- We are interested in a steady-state solution,
  $\Rightarrow$ omit the time-derivatives

- Rewrite the system in residual form, i.e.

$$\mathbf{r}(\mathbf{s}) = \begin{pmatrix} (r_1, r_2, r_3)^T \\ r_4 \\ r_5 \end{pmatrix} = \begin{pmatrix} (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p - \nabla \cdot [2\nu D(\mathbf{u})] \\ -\nabla \cdot \mathbf{u} \\ (\mathbf{u} \cdot \nabla) T - \nabla(\alpha \cdot \nabla T) \end{pmatrix}$$

with state vector $\mathbf{s} = (\mathbf{u}, p, T)^T$

Lagrange viewpoint

- Introduce Lagrange function/multipliers and transform into an unconstrained optimization problem:

$$L(\mathbf{s}, \beta) \quad = \quad I(\mathbf{s}, \beta) - \lambda^T \mathbf{r}(\mathbf{s}, \beta)$$

- Considering general variation $\delta \mathbf{s}$ and $\delta\beta$ gives

$$\delta L \quad = \quad \left(\frac{\partial I}{\partial \mathbf{s}} - \lambda^T \frac{\partial \mathbf{r}}{\partial \mathbf{s}} \right) \delta \mathbf{s} + \left(\frac{\partial I}{\partial \beta} - \lambda^T \frac{\partial \mathbf{r}}{\partial \beta} \right) \delta\beta$$

- If λ^T is chosen to satisfy the adjoint equation

$$\frac{\partial I}{\partial \mathbf{s}} - \lambda^T \frac{\partial \mathbf{r}}{\partial \mathbf{s}} = 0 \quad \Rightarrow \quad \left(\frac{\partial \mathbf{r}}{\partial \mathbf{s}} \right)^T \lambda = \left(\frac{\partial I}{\partial \mathbf{s}} \right)^T$$

we obtain

$$\delta L \quad = \quad \left(\frac{\partial I}{\partial \beta} - \lambda^T \frac{\partial \mathbf{r}}{\partial \beta} \right) \delta\beta$$

Deriving the adjoint system

Starting point: vanishing variation of the Lagrange function:

$$
\begin{aligned}
\delta_{\mathbf{s}} L &= \sum_i \left(\int_\Omega \frac{\partial I_\Omega}{\partial s_i} \delta s_i \,\mathrm{d}\Omega + \int_\Gamma \frac{\partial I_\Gamma}{\partial s_i} \delta s_i \,\mathrm{d}\Gamma \right) + \sum_{i,j} \int_\Omega \hat{s}_i \frac{\partial r_j}{\partial s_i} \delta s_i \,\mathrm{d}\Omega \\
&= \int_\Omega \frac{\partial I_\Omega}{\partial \mathbf{u}} \delta \mathbf{u} \,\mathrm{d}\Omega + \int_\Gamma \frac{\partial I_\Gamma}{\partial \mathbf{u}} \delta \mathbf{u} \,\mathrm{d}\Gamma + \int_\Omega \hat{s} \cdot \delta_{\mathbf{u}} \mathbf{r} \,\mathrm{d}\Omega \\
&\quad + \int_\Omega \frac{\partial I_\Omega}{\partial p} \delta p \,\mathrm{d}\Omega + \int_\Gamma \frac{\partial I_\Gamma}{\partial p} \delta p \,\mathrm{d}\Gamma + \int_\Omega \hat{s} \cdot \delta_p \mathbf{r} \,\mathrm{d}\Omega \\
&\quad + \int_\Omega \frac{\partial I_\Omega}{\partial T} \delta T \,\mathrm{d}\Omega + \int_\Gamma \frac{\partial I_\Gamma}{\partial T} \delta T \,\mathrm{d}\Gamma + \int_\Omega \hat{s} \cdot \delta_T \mathbf{r} \,\mathrm{d}\Omega \equiv 0.
\end{aligned}
$$

with $\hat{\mathbf{s}} = (\hat{\mathbf{u}}, \hat{p}, \hat{T})^T$ adjoint state variables (Lagrange multiplier)

Variation of the residual form

After several basic transformation

$$
\begin{aligned}
\delta_{\mathbf{s}} L &= \int\limits_{\Omega} \begin{pmatrix} \delta \mathbf{u} \\ \delta p \\ \delta T \end{pmatrix} \cdot \begin{pmatrix} -\nabla \hat{\mathbf{u}} \cdot \mathbf{u} - (\mathbf{u} \cdot \nabla)\hat{\mathbf{u}} - \nabla \cdot (2\nu \mathbf{D}(\hat{\mathbf{u}})) + \nabla \hat{p} - T \nabla \hat{T} & + \\ -\nabla \cdot \delta \mathbf{u} & + \\ (\delta \mathbf{u} \cdot \nabla) T & + \end{pmatrix} \\
&+ \int\limits_{\Gamma} \begin{pmatrix} \delta \mathbf{u} \\ \delta p \\ \delta T \end{pmatrix} \cdot \begin{pmatrix} \mathbf{n}(\hat{\mathbf{u}} \cdot \mathbf{u} + \hat{\mathbf{u}}(\mathbf{u} \cdot \mathbf{n}) + 2\nu \mathbf{n} \cdot \mathbf{D}(\hat{\mathbf{u}}) + T \hat{T} \mathbf{n} - \hat{p} \mathbf{n} & + \frac{\partial I_\Gamma}{\partial \mathbf{u}} \\ \hat{\mathbf{u}} \cdot \mathbf{n} & + \frac{\partial I_\Gamma}{\partial p} \\ \nu \mathbf{n} \cdot \nabla \hat{T} + \hat{T}(\mathbf{u} \cdot \mathbf{n}) & + \frac{\partial I_\Gamma}{\partial T} \end{pmatrix} \\
&+ \int\limits_{\Gamma} \begin{pmatrix} \mathbf{u} \\ p \\ T \end{pmatrix} \cdot \begin{pmatrix} -2\nu \mathbf{n} \cdot \mathbf{D}(\delta \mathbf{u}) \\ 0 \\ -\nu \mathbf{n} \cdot (\delta T) \end{pmatrix} \mathrm{d}\Gamma \\
&= \int\limits_{\Omega} \delta \mathbf{s} \cdot \left(\hat{\mathbf{r}} + \frac{\partial I}{\partial_{\mathbf{s}}} \right) \mathrm{d}\Omega + \int\limits_{\Gamma} \delta \mathbf{s} \cdot \hat{\mathbf{b}}_{c_1} \mathrm{d}\Gamma + \int\limits_{\Gamma} \mathbf{s} \cdot \hat{\mathbf{b}}_{c_2} \mathrm{d}\Gamma
\end{aligned}
$$

Adjoint system

The corresponding inhomogeneous adjoint system
(Time-independent incompressible adjoint N.-S. with heat diffusion)
to the optimization problem is $\hat{\mathbf{r}} = \frac{\partial I}{\partial_{\mathbf{s}}}$ i.e.

$$
\begin{aligned}
\mathbf{D}(\hat{\mathbf{u}})\mathbf{u} + \nabla \cdot (2\nu \mathbf{D}(\hat{\mathbf{u}})) + \nabla \hat{p} - T \nabla \hat{T} &= \frac{\partial I_\Omega}{\partial \mathbf{u}} \\
\nabla \cdot \hat{\mathbf{u}} &= \frac{\partial I_\Omega}{\partial p} \\
\mathbf{u} \cdot \nabla \hat{T} + \nabla \cdot (\nu \nabla \hat{T}) &= \frac{\partial I_\Omega}{\partial T}
\end{aligned}
$$

for full derivation:

Hinterberger, C., Olesen, M., Industrial application of continuous adjoint flow solvers for the optimization of automotive exhaust systems. *Proc.ECCOMAS-CFD&Optimization*, 2011.

Adjoint Boundary conditions

For the boundary integrals we have to fulfil the following expression:

$$
\begin{aligned}
0 &\equiv \delta \mathbf{s} \cdot \hat{\mathbf{b}}_{c_1} + \mathbf{s} \cdot \hat{\mathbf{b}}_{c_2} \\
&= \begin{pmatrix} \delta \mathbf{u} \\ \delta p \\ \delta T \end{pmatrix} \cdot \begin{pmatrix} \mathbf{n}(\hat{\mathbf{u}} \cdot \mathbf{u} + \hat{\mathbf{u}}(\mathbf{u} \cdot \mathbf{n}) + 2\nu \mathbf{n} \cdot \mathbf{D}(\hat{\mathbf{u}}) + T \hat{T} \mathbf{n} - \hat{p} \mathbf{n} + \frac{\partial I_{\Gamma}}{\partial \mathbf{u}} \\ \hat{\mathbf{u}} \cdot \mathbf{n} + \frac{\partial I_{\Gamma}}{\partial p} \\ \nu \mathbf{n} \cdot \nabla \hat{T} + \hat{T}(\mathbf{u} \cdot \mathbf{n}) + \frac{\partial I_{\Gamma}}{\partial T} \end{pmatrix} \\
&+ \begin{pmatrix} \mathbf{u} \\ p \\ T \end{pmatrix} \cdot \begin{pmatrix} -2\nu \mathbf{n} \cdot \mathbf{D}(\delta \mathbf{u}) \\ 0 \\ -\nu \mathbf{n} \cdot (\delta T) \end{pmatrix}, \qquad \forall \delta \mathbf{s}, \mathbf{s}.
\end{aligned}
$$

Thus, BCs depend on objective function

Adjoint boundary conditions

With $\hat{\mathbf{u}} = \hat{\mathbf{u}}_t + \hat{\mathbf{u}}_n = u_t\mathbf{t} + u_n\mathbf{n}$ and $\mathbf{t} \perp \mathbf{n}$ we derive adjoint BCs:

Inlet

$$\hat{\mathbf{u}}_t = \mathbf{0}, \quad \hat{u}_n = -\frac{\partial I_\Gamma}{\partial p}, \quad \frac{\partial \hat{p}}{\partial \mathbf{n}} = 0, \quad \hat{T} = 0.$$

Wall

$$\hat{\mathbf{u}}_t = \mathbf{0}, \quad \hat{u}_n = -\frac{\partial I_\Gamma}{\partial p}, \quad \frac{\partial \hat{p}}{\partial \mathbf{n}} = 0, \quad \frac{\partial \hat{T}}{\partial \mathbf{n}} = -\frac{1}{\alpha}\frac{I_\Gamma}{\partial T}.$$

Outlet

$$u_n\hat{\mathbf{u}}_t + \nu(\mathbf{n}\cdot\nabla)\hat{\mathbf{u}}_t = \frac{\partial I_\Gamma}{\partial \mathbf{u}_t},$$

$$\hat{\mathbf{u}}\cdot\mathbf{u} + \hat{u}_n u_n + \nu(\mathbf{n}\cdot\nabla)\hat{u}_n + T\hat{T} + \frac{\partial I_\Gamma}{\partial \mathbf{u}_n} = \hat{p}$$

$$u_n\hat{T} + \alpha\frac{\partial \hat{T}}{\partial \mathbf{n}} = \frac{\partial I_\Gamma}{\partial T}$$

Objective function

- We see that the BCs depends on the cost function.
- In our case, we focus on maximizing

Heat conduction on the wall

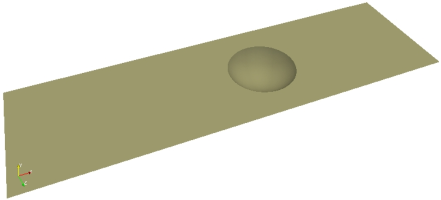
$$I = \int_{wall} \frac{\partial T}{\partial \mathbf{n}} \mathrm{d}\Gamma$$

May be problematic:

$$\frac{\partial I_\Gamma}{\partial T} = \frac{\partial}{\partial T}\left(\frac{\partial T}{\partial \mathbf{n}}\right) = 0$$

- Influence of the cost function on to the BCs?
- Alternative formulation (in terms of enthalpy)?

Test case

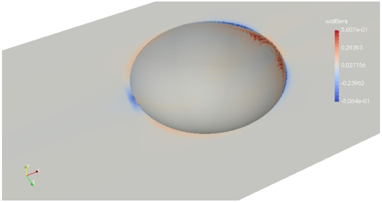


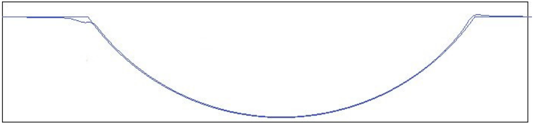
- Rectangular domain with
  - length $x = 0.0, 0.276\,\text{m}$
  - width $y = 0.08\,\text{m}$
  - height $z = 0.03\,\text{m}$.
- Dimple: diameter $\text{d} = 0.048\,\text{m}$. and height $\text{h} = 0.012\,\text{m}$.

Test case | Shape update

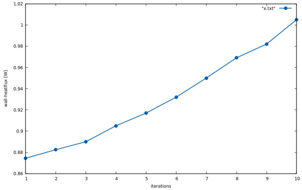
Local surface sensitivity of a normal displacement of the surface:

$$\frac{\partial L}{\partial \beta_i} \approx -\nu \frac{\partial \mathbf{u}_t}{\partial \mathbf{n}_i} \frac{\partial \hat{\mathbf{u}}_t}{\partial \mathbf{n}_i} - \alpha \frac{\partial T}{\partial \mathbf{n}_i} \frac{\partial \hat{T}}{\partial \mathbf{n}_i}$$

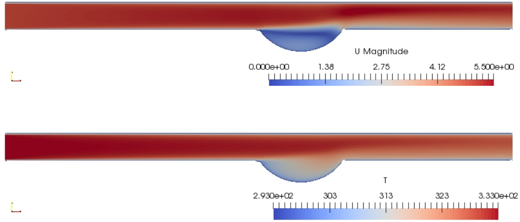




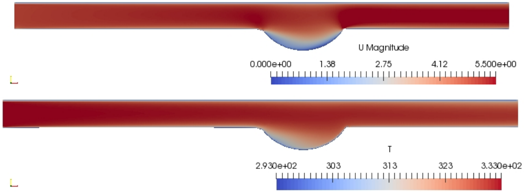
Updated vs. initial shape



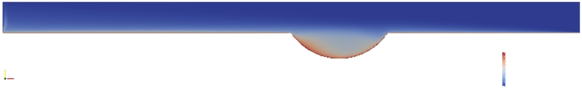
Test case | Velocity and temperature (primal)



Initial(above) optimised(below)

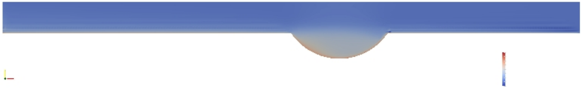


Test case | Adjoint fields



Adjoint Temperature

Adjoint Velocity



Adjoint Pressure

Outlook

So far

- Proof of concept.

Next steps

- Validation
- Consolidation

Further plans

- Improved mesh deformation
  - RB functions
  - Free Form Deformation (FFD) techniques
- Combination with other cost functions
  - pressure loss
  - uniformity
- Alternative cost function formulation?